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CRT STUDY MATERIAL FOR

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DADI INSTITUTE OF ENGINEERING AND TECHNOLOGY, ANAKAPALLI

QUANTITATIVE APTITUDE
FOR
CAMPUS RECRUITMENT TRAINING
(CRT)

CONTENTS

PIPES AND CISTERNS

Pipes: Generally the pipes are connected to tank or cistern and are used to fill or empty the tank.

Inlet: A pipe connected with a tank or a cistern that fills it is known as inlet.

Outlet: A pipe connected with a tank or cistern emptying it is known as outlet.

Note:

1. Pipes and Cistern problems are similar to those on Time and Work.
2. The only difference here is, the work done is in terms of filling or emptying a cistern and the time taken by a pipe or leak (Crack) to fill or empty a cistern respectively.
3. Generally, the time taken to fill a cistern is taken as positive and the time taken to empty a cistern is taken as negative.

4. The amount of work done i.e., filling or emptying a cistern is generally taken as unity, unless otherwise specified.

Formulae:

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1. If an inlet can completely fill the empty tank in X hours, the part of the tank filled in 1 hour = $\frac{1}{X}$.

2. If an outlet can empty the full tank in Y hours, the part of the tank emptied in 1 hour = $\frac{1}{Y}$.

3. If both inlet and outlet are open, net part of the tank filled in 1 hour = $\frac{1}{X} - \frac{1}{Y}$.

4. Two pipes A and B can fill or empty a cistern in X and Y hours respectively, while working alone. If both the pipes are opened together, then the time taken to fill or empty the cistern = $\frac{XY}{X+Y} hours.$

5. Three pips A, B and C can fill a cistern in X, Y and Z hours respectively, while working alone. If all the three pipes are opened together, the time taken to fill the cistern = $\left(\frac{XYZ}{XY+YZ+ZX}\right) hours.$

   Note: This type of formulae can be generated by replacing negative sign wherever a pipe starts emptying the cistern instead of the standard positive sign.

6. Two pipes A and B can fill a cistern in X hours and Y hours, respectively. There is also an outlet C. if all the three pipes are opened together, the tank is full in Z hours. The time taken by C to empty thefull tank = $\left(\frac{XYZ}{YZ+ZX-XY}\right) hours.$

7. A tank takes X hours to be filled by a pipe. But due to a leak, it is filled in Y hours. The amount of time in which the leak can empty the full tank = $\left(\frac{XY}{Y-X}\right) hours.$

8. A cistern has a leak which can empty it in X hours. A pipe which allows Y litres of water per hour into the cistern is turned on and now the cistern is emptied in Z hours. The capacity of the cistern is $\left(\frac{XYZ}{Z-X}\right) litres.$

9. One fill pipe A is k times faster than the other fill pipe B.

   a) If B can fill a cistern in x hours, then the time in which the cistern will be full, if both the full pipes are opened together, is $\left(\frac{x}{k+1}\right) hours.$

   b) If A can fill a cistern in y hours, then the time in which the cistern will be full, if both the full pipes are opened together, is $\left(\frac{k}{k+1}\right) \times y hours.$

10. If one fill pipe A is k times faster and takes x minutes less time than the other fill pipe B, then

    a) A will fill the cistern in $\left(\frac{x}{k-1}\right)$ minutes.

    b) B will fill the cistern in $\left(\frac{kx}{k-1}\right)$ minutes.

    c) The time taken to fill a cistern, if both the pipes are opened together is $\left(\frac{kx}{(k-1)^2}\right)$ minutes.

Solved Examples

1. A pipe can fill a tank in 4 hours. Find the part of tank filled in one hour.

Sol: The part of tank filled in 1 hour = $\frac{1}{4}$.

2. A pipe can fill a tank in 20 minutes. Find the time in which $\frac{1}{5}$ part of the tank will be filled.

Sol: The part of the tank filled in 1 minute = $\frac{1}{20}$

So, $\frac{1}{5}$ part of the tank is filled in $20 \times \frac{1}{5}$ = 4 minutes.

3. A pipe can empty a cistern in 20 minutes. Find the time in which $\frac{2}{5}$ part of the cistern will be emptied.

Sol: $\frac{2}{5}$ part of the cistern is emptied in $20 \times \frac{2}{5} = 8$ minutes.

4. A pipe can empty a cistern in 20 hours. Find the part of the cistern emptied in 4 hours.

Sol: The part of the cistern emptied in 1 hour = $\frac{1}{20}$

So, the part of the cistern emptied in 4 hours = $4 \times \frac{1}{20} = \frac{1}{5}$.

5. A tap can fill a cistern in 8 hours and another can empty it in 12 hours. If both the taps are opened simultaneously, find the time in hours to fill the cistern.

Sol: Here, X = 8 and Y = 12

Part of the cistern filled in 1 hour = $\frac{1}{X} - \frac{1}{Y} = \frac{1}{8} - \frac{1}{12} = \frac{1}{24}$.

Total time taken to fill the cistern = 24 hours.

6. Two pipes A and B can fill a cistern in 20 and 30 minutes, respectively. If both the pipes are simultaneously then find how long will it take to fill the cistern?

Sol: Here, X = 20 and Y = 30

Part of the cistern filled by (A + B) in 1 minute =

$$\frac{1}{20} + \frac{1}{30} = \frac{1}{20} + \frac{1}{30} = \frac{5}{60} = \frac{1}{12}$$

Hence, both the pipes together will fill the cistern in 12 minutes.

7. Two pipes A and B can separately fill a cistern in 4 hours and 6 hours respectively, while a third pipe C can empty it in 3 hours. In what time will the cistern be full, if all the pipes are opened together?

Sol: Here, X = 4, Y = 6 and Z = - 3

So, the cistern will be full in $\left(\frac{4 \times 6 \times -3}{(4 \times 6) + (6 \times -3) + (4 \times -3)}\right) = \frac{72}{6} = 12$ hours.

8. Two taps A and B can fill a cistern in 15 and 30 minutes respectively. There is a third exhaust tap C at the bottom of tank. If all taps are opened at the same time, the cistern will be full in 25 minutes. In what time can exhaust tap C empty the cistern when full?

Sol: Here, X = 15, Y = 30 and Z = 25

C can empty the full tank in $\left(\frac{XYZ}{YZ + ZX - XY}\right)$

$$= \left(\frac{15 \times 30 \times 25}{(15 \times 30) + (30 \times 45) - (15 \times 45)}\right) \text{minutes}$$

$$= \frac{15 \times 30 \times 25}{1125} = 10 \text{ minutes.}$$

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9. A pipe can fill a tank in 6 hours. Due to leakage in the bottom, it is filled in 12 hours. If the tank is full, how much time will the leak take to empty it?

Sol: Here, X = 6 and Y = 12

So, the time taken by the leak to empty the full tank = $\left(\frac{XY}{Y-X}\right) hours$

$$= \left(\frac{6 \times 12}{12-6}\right) = 12 hours.$$

10. A leak in the bottom of a tank can empty the full tank in 3 hours. An inlet pipe fills water at the rate of 2 litres per minute. When the tank is full, the inlet is opened and due to leak, the tank is empty in 5 hours. Find the capacity of the tank.

Sol: Here, X = 3, Y = 2 x 60 = 120 and Z = 5

So, the capacity of the tank = $\left(\frac{XYZ}{Z-X}\right) litres = \left(\frac{3 \times 120 \times 4}{5-3}\right) = 720$ litres

11. One fill pipe A is 2 times faster the second fill pipe B. If A can fill a cistern in 9 minutes, then find the time when the cistern will be full if both fill pipes are opened together.

Sol: Here, k = 2 and y = 9

So, Cistern will be full in $\left(\frac{k}{k+1}\right) \times y = \frac{2}{2+1} \times 9 = 6$ minutes.

12. One fill pipe A is 7 times faster the second fill pipe B takes 72 minutes less than the fill pipe B. When will the cistern be full if both fill pipes are opened together?

Sol: Here, k = 7 and x = 72

Cistern will be full in $\left(\frac{kx}{(k-1)^2}\right) = \frac{7 \times 72}{(7-1)^2} = 14$ minutes.

Exercise - 9

1. A alone can complete the work in 12 days while A and B together can complete the same work in 8 days. The number of days that B will take to complete the work alone is ____________
   1) 10 2) 24
   3) 20 4) 9

2. A can do a work in 6 days and B in 9 days. How many days will both take together to complete the work.
   1) 7.5 2) 5.4
   3) 3.6 4) 3

3. A can do a piece of work in 4 hours, B and C can do it in 3hrs, A and C can do it in 2hrs. How long will B alone take to do it?
   1) 10hrs 2) 12hrs
   3) 8hrs 4) 24hrs

4. 10 men and 15 women finish a work in 6 days. One man alone finishes that work in 100 days. In how many days will a woman finish the work?
   1) 125 2) 150
   3) 90 4) 225

5. A completes a work in 12 days; B completes the some work in 15 days. A started working alone and after 3 days B joined him. How many days will they now take together to complete the remaining work?
   1) 5 2) 8
   3) 6 4) 4

6. 10 men can complete a piece of work in 15 days & 15 women can complete the same work in 12 days. If all the 10 men & 15 women work together, in how many days will the work get completed?
   1) 6 2) $7\frac{2}{3}$
   3) $6\frac{2}{3}$ 4) None of these

7. A can do a certain work in the same time in which B & C together can do it. If A and B together could do it in 10 days and C alone in 50 days then B alone could do the work in
   1) 15 days 2) 20 days
   3) 25 days 4) 30 days

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8. A& B under took to do a piece of work for Rs.4,500. A alone could do it in 8 days and B alone in 12 days. With the assistance of C they finished the work in 4 days. Then C's share of the money is ______________
   1) Rs.2,250 2) Rs.1,500
   3) Rs.750 4) Rs.375

9. A can finish a work in 24 days, B in 9 days and C in 12 days. B & C start the work but are forced to leave after 3 days. The remaining work is done by A in ______________
   1) 5 days 2) 6 days
   3) 10 days 4) $10\frac{1}{2}$ days

10. If 3 men (or) 4 women can plough a field in43 days, how long will 7 men and 5 women take to plough it.
   1) 10 days 2) 11 days
   3) 9 days 4) 12 days

11. A can do $\frac{3}{4}^{th}$ of a work in 12 days. In how many days can he finish $\frac{1}{8}^{th}$ of work?
   1) 1 day 2) 2 days
   3) 4 days 4) 8 days

12. If 72 men can build a wall 280m. long in 21 days, how many men will take 18 days to build a similar type of wall of length 100m.?
   1) 30 2) 10
   3) 18 4) 28

13. A takes twice as much time as B or thrice as much time as C to finish a piece of work. Working together, they can finish the work in 2 days. B can do the work alone in
   1) 12 days 2) 4 days
   3) 8 days 4) 6 days

14. A does $\frac{4}{5}$ of a piece of work in 20 days; he then calls in B and they finish the remaining work in 3 days. How long will B alone take to do the whole work?
   1) $37\frac{1}{2}$ days 2) 37 days
   3) 40 days 4) 23 days

15. A does half as much work as B in 1/6 of the time. If together they take 10 days to complete a work, how many days shall B take to do it alone?
   1) 15 days 2) 30 days
   3) 40 days 4) 50 days

16. A man, a woman and a boy can together complete a piece of work in 3 days. If a man alone can do it in 6 days and a boy alone can do it in 18 days, how long will a woman alone take to complete the work.
   1) 9 days 2) 21 days
   3) 24 days 4) 27 days

17. If the wages of 6 men for 15 days be Rs.700, then the wages of 9 men for 12 days will be ____________
   1) Rs.700 2) Rs.840
   3) Rs.1050 4) Rs.900

18. A man is paid Rs.20 for each day he works, and forfeits Rs.3 for each day he is idle. At the end of 60 days he gets Rs.280. Then he was idle for ______________
   1) 20 days 2) 25 days
   3) 30 days 4) 40 days

19. A team of 10 men can complete a particular job in 12 days. A team of 10 women can complete the same job in 6 days. How many days are needed to complete the job if the two teams work together?
   1) 4 2) 6
   3) 9 4) 18

20. A contractor undertook to finish a certain work in 124 days and employed 120 men on it. After 64 days, he found that he had already done $\frac{2}{3}^{rd}$ of the work. How many men he can discharge now so that the work may finish in time

1) 24 2) 56
3) 64 4) 80

21. A work could be completed in 100 days. However, due to the absence of 10 workers, it was completed in 110 days. The original number of workers was ____________
    1) 100 2) 110
    3) 55 4) 50

22. A contractor under takes to make a road in 40 days and employs 25 men. After 24 days, he finds that only one-third of the road is made. How many extra men should he employ so that he is able to complete the work 4 days earlier?
    1) 100 2) 60
    3) 75 4) none of these

23. 30 men complete one third of a work in 30 days. How many more men should be employed to finish the rest of the work in 40 more days?
    1) 15 2) 45
    3) 20 4) 25

24. A and B under took to do a piece of work for Rs.900. A alone could do it in 60 days and B in 30 days. If A & B work together and complete the work, then the share of B ________
    1) Rs.600 2) Rs.400
    3) Rs.300 4) Rs.200

25. 5 men or 6 women or 10 boys can do a work in 15 days. How long will it take to complete the work by a group of 5 men, 6 women and 10 boys?
    1) 5 days 2) 6 days
    3) 10 days 4) 45 days

26. A can do a piece of work in 30 days. B in 15 days and C in 10 days. They started the work all together but B put $\frac{1}{2}$ time daily and C put $\frac{1}{3}$ time daily to help A in doing the work. The work will last in _______________
    1) 30 days 2) 10 days
    3) 20 days 4) 25 days

27. A can do a work in 15 days & B the same work in 12 days. B started the work and was joined by A, 5 days before the end of work. The work lasted for______days.
    1) 8 2) 12
    3) 13 4) 24

28. A and B can do a piece of work in 40 days while C & A can do it in 60 days. If B is twice as good as C, then C alone will do the work in____________days.
    1) 120 2) 100
    3) 80 4) 24

29. A hostel has provision for 800 men for 24 days at the rate of 2 kg per man per day. For how many men is the provision sufficient, for 20 days at the rate of 1.5 kg per man per day?
    1) 1280 2) 1000
    3) 1820 4) 1240

30. 12 men can do a work in 15 days working 8 hours a day. In how many days can 9 men do the same work, working 10 hours a day?
    1) 10 2) 16
    3) 18 4) 24

31. Two taps A and B can separately fill a tank in 20 and 30 hours respectively. If both the pipes are opened simultaneously, how much time will be taken to fill the tank?
    1) 10 hrs 2) 11 hrs
    3) 18 hrs 4) 12 hrs

32. A tap can fill a tank in 12 minutes and another tap in 15 minutes, but a third tap can empty it in 6 minutes. The three taps are kept open together. Find when the cistern is emptied or filled?
    1) 60 min. to fill 2) 30 min. to fill
    3) 60 min to empty 4) 30 min to empty

33. Two taps A & B can fill a cistern in 12 and 16 minutes respectively. Both fill taps are opened together, but 4 minutes before cistern is full, one tap A is closed. How much time will the cistern take to fill?
    1) $9\,{}^{1}/_{7}$ min. 2) $3\,{}^{1}/_{7}$ min.
    3) $11\,{}^{1}/_{7}$ min. 4) None.

34. A ship 55 km from the shore springs a leak which admits 2 tonnes of water in 6 minutes. 80 tonnes would suffer to sink her, but the pumps can throw out 12 tonnes an hour. Find the average rate of sailing that she may just reach the shore as she begins to sink.
    1) 5.5 kmph 2) 2.5 kmph
    3) 1.8 kmph 4) 4 kmph

35. A tap can fill a swimming pool in h hours. What part of the pool is filled in y hours?

h

1) yh 2) y

3) $\frac{y}{h}$ 2 2 4) h – y

TIME AND DISTANCE

Speed: The speed of an object is the distance covered by it in a unit time interval. It is obtained by dividing the distance covered by the object, by the time it takes to cover the distance.

Thus, Speed = $\frac{\text{Distance travelled}}{\text{time taken}}$

Note:

1. The terms, 'time' and 'distance' are related to the speed of a moving object.
2. If the time taken is constant, the distance travelled is proportional to the speed i.e. more the speed; more the distance travelled in the same time.
3. If the speed is constant, the distance travelled is proportional to the time taken i.e. more the distance travelled; more the time taken at the same speed.
4. If the distance travelled is constant, the speed is inversely proportional to the time taken i.e. more the speed; less the time taken for the same distance travelled.

Formulae:

1. Speed = $\frac{\text{Distance}}{\text{Time}}$
2. Distance = Speed x Time
3. Time = $\frac{\text{Distance}}{\text{Speed}}$
4. x km/hr = $\left(x\times\frac{15}{8}\right)$ m/sec
5. x m/sec = $\left(x\times\frac{8}{15}\right)$ km/hr
6. If A covers a distance d_1 km at a speed s_1 km/hr and then d_2 km at s_2 km/hr, then the average speed during the whole journey is

   Average speed = $\frac{s_1 s_2(d_1+d_2)}{s_1 d_2+s_2 d_1}$ km/hr
7. If A goes from X to Y at s_1 km/hr and comes back from Y to X at s_2 km/hr, then the average speed during the whole journey is

   Average speed = $\frac{2s_1 s_2}{s_1+s_2}$ km/hr
8. A goes from X to Y at s_1 km/hr and returns back from Y to X at s_2 km/hr. If he takes T hours in all, the distance between A and B is $T\left(\frac{s_1 s_2}{s_1+s_2}\right)km.$
9. A and B start at the same time from two points P and Q towards each other and after crossing they take T_1 and T_2 hours in reaching Q and P respectively, then speed

   $$\frac{\text{A's speed}}{\text{B's speed}}=\frac{\sqrt{T_2}}{\sqrt{T_1}}$$
10. If a body travels d_1, d_2, d_3, ….., d_n metres with different speeds s_1, s_2, s_3,………., s_n m/sec in time T_1, T_2, T_3, ………, T_n seconds respectively, then the average speed of the body throughout the journey is given by

    a) If d_1, d_2, …….., d_n and T_1, T_2, ……., T_n are known

    $$V_a=\frac{\text{Total distance travelled}}{\text{Total time taken}}=\frac{d_1+d_2+d_3+.......+d_n}{T_1+T_2+T_3+.........+T_n}$$

b) If $d_1, d_2, \ldots\ldots, d_n$ and $s_1, s_2, \ldots\ldots, s_n$ are known

$$V_a = \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{s_1T_1 + s_2T_2 + \ldots\ldots\ldots + s_nT_n}{T_1 + T_2 + \ldots\ldots\ldots + T_n}$$

11. If the new speed is $\frac{a}{b}$ of the original speed, then the change in time taken to cover the same distance is given by

$$\text{Change in time} = \left(\frac{b}{a} - 1\right) \times \text{original time}$$

12. A body covers a distance d in time T_1 with speed s_1, but when it travels with speed s_2 covers the same distance in time T_2 then

$$\frac{\text{Product of speed}}{d} = \frac{s_1}{T_2} = \frac{s_2}{T_1} = \frac{\text{Difference of speed}}{\text{Difference of time}}$$

Note: By equating any two of the above, we can find the unknowns as per the given question.

13. A train travels a certain distance at a speed of s_1 km/hr without stoppings and it covers the same distance at a speed of s_2 km/hr with stoppings then

$$\text{The stopping time per hour} = \frac{\text{Difference of speed}}{\text{speed without stoppings}} = \left(\frac{s_1 - s_2}{s_1}\right) hr.$$

14. If a train overtakes a pole or a man or a milestone, then the distance covered in overtaking = Length of the train.

15. If a train overtakes a bridge or tunnel or a platform or another train, then the distance covered = Sum of the two lengths.

16. Relative Speed: If two trains of lengths L_1 km and L_2 km, respectively are traveling in the same direction at s_1 km/hr and s_2 km/hr respectively such that $s_1 > s_2$, then their

    a) Relative speed = $s_1 - s_2$.

    b) Time taken by the faster train to cross the slower train = $\left(\frac{L_1 + L_2}{s_1 - s_2}\right)$ hr.

17. Relative Speed: If two trains of lengths L_1 km and L_2 km, respectively are traveling in the opposite direction at s_1 km/hr and s_2 km/hr respectively then their

    a) Relative speed = $s_1 + s_2$.

    b) Time taken by the faster train to cross each other = $\left(\frac{L_1 + L_2}{s_1 + s_2}\right)$ hr.

18. Two trains of lengths L_1 m and L_2 m run on parallel tracks. When running in the same direction, the faster train passes the slower one in T_1 seconds, but when they are running in opposite directions with the same speeds as earlier, they pass each other in T_2 seconds then

    a) Speed of the faster train = $\frac{L_1 + L_2}{2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right)$ m/s.

    b) Speed of the slower train = $\frac{L_1 + L_2}{2}\left(\frac{1}{T_1} - \frac{1}{T_2}\right)$ m/s.

19. A train starts from a place at s_1 km/hr and another fast train starts from the same place after T hours at s_2 km/hr in the same direction. Then,

    a) The distance from the starting place at which both the trains will meet is given by $\left(\frac{s_1 \times s_2 \times T}{s_2 - s_1}\right)$ km.

    b) The time after which the two trains will meet is given by $\left(\frac{s_1T}{s_2 - s_1}\right)$ hr.

20. The distance between two stations A and B is d km. A train starts from A to B at s_1 km/hr. T hours later another train starts from B to A at s_2 km/hr. Then,

    a) The distance from the A at which both the trains will meet is given by $s_1\left(\frac{d + s_2T}{s_1 + s_2}\right)$ km.

b) The time after which the two trains will meet is given by $\left(\dfrac{d + s_2 T}{s_1 + s_2}\right)$ hr.

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21. Two trains start simultaneously from the stations A and B towards each other each other with speed s_1 km/hr and s_2 km/hr, respectively. When they meet it is found that the second train had travelled d km more than the first. Then the

Distance between two trains = $d\left(\dfrac{s_1 + s_2}{s_2 - s_1}\right)$ km.

Solved Examples

1. Find the speed of a train which covers a distance of 160 km in 4 hours.

Sol: Speed = $\dfrac{\text{Distance}}{\text{Time}} = \dfrac{160}{4} = 40\text{km/hr.}$

2. How long does a train 200 m long running at the rate of 80 km/hr take to cross a telegraphic pole?

Sol: We know, in crossing the telegraphic pole, the train must travel its own length.
So, Distance travelled = 200 m

Speed = 80 km/hr = $\dfrac{80 \times 1000}{60 \times 60}$ m/sec $= \dfrac{200}{9}$ m/sec

So, time taken to cross the pole = $\dfrac{200}{\frac{200}{9}} = 9$ seconds.

3. A train running at a speed of 72 km/hr passes a pole on the platform in 15 seconds. Find the length of the train in metres.

Sol: Speed of the train = 72 km/hr = $72 \times \dfrac{5}{18} = 20\text{m/sec}$

Length of the train = Speed of the train x time taken
= 20 x 15 = 300 m.

4. A ship sails to Vizag at a speed of 10 knots/hr and sails back to the same point at the rate of 15 knots/hr. Find the average speed for the whole journey.

Sol: Here s_1 = 10 and s_2 = 15

Average speed = $\dfrac{2s_1 s_2}{s_1 + s_2}$ km/hr = $\dfrac{2 \times 10 \times 15}{10 + 15} = 12\text{knots/hr.}$

5. Sheela started to a bakery with the speed of 5 km an hour and returns with a speed of 3 km/hr. if she takes 8 hours in all, find the distance in km between the bakery and her house.

Sol: Here s_1 = 5, s_2 = 3 and T = 8

The distance between the bakery and her house = $T\left(\dfrac{s_1 s_2}{s_1 + s_2}\right)$

$= 8 \times \left(\dfrac{5 \times 3}{5 + 3}\right) = 15\text{km}$.

6. Sujay starts his journey from Bombay to Kolkata and simultaneously Niteesh starts from Kolkata to Bombay. After crossing each other they finish their remaining journey in $6\frac{1}{4}$ hours and 4 hours respectively. What is Niteesh's speed if Sujay's speed is 40 km/hr?

Sol: $\dfrac{\text{Sujay's speed}}{\text{Niteesh's speed}} = \sqrt{\dfrac{T_2}{T_1}} = \dfrac{\sqrt{4}}{\sqrt{6\frac{1}{4}}} = \dfrac{\sqrt{4}}{\sqrt{\frac{25}{4}}} = \sqrt{\dfrac{16}{25}} = \dfrac{4}{5}$

So, Niteesh's speed = $\dfrac{5}{4} \times \text{Sujays speed} = \dfrac{5}{4} \times 40 = 50\text{km/hr.}$

7. A bike during its journey travels 20 min at a speed of 15 km/hr, another 25 min at 30 km/hr and another 30 min at 15 km/hr. Find the average speed of the bike.

20 25 30

Sol: $T_1 = \frac{}{60}$, $T_2 = \frac{}{60}$, $T_3 = \frac{}{60}$, s_1 = 15, s_2 = 30 and s_3 = 15

So, Average speed of the bike = $\dfrac{s_1T_1 + s_2T_2 + s_3T_3}{T_1 + T_2 + T_3}$

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$$= \frac{15\times\frac{20}{60} + 30\times\frac{25}{60} + 15\times\frac{30}{60}}{\frac{20}{60} + \frac{25}{60} + \frac{30}{60}}$$

$$= \frac{15\times\frac{20}{60} + 30\times\frac{25}{60} + 15\times\frac{30}{60}}{\frac{20}{60} + \frac{25}{60} + \frac{30}{60}}$$

$$= \frac{300 + 750 + 450}{75} = \frac{1500}{75} = 20\text{km/hr.}$$

8. By walking at $\frac{3}{4}$ of her usual speed, Rani is 5 minutes late to the conference. Find her usual time to cover the distance.

Sol: Here, change in time = 5 and $\frac{a}{b} = \frac{3}{4}$

We have, change in time = $\left(\frac{b}{a} - 1\right)\times \text{original time}$

$\Rightarrow$ original time = $\dfrac{\text{change in time}}{\left(\frac{b}{a} - 1\right)} = \dfrac{5}{\left(\frac{4}{3} - 1\right)} = 15 \text{ minutes.}$

9. Two scooterists do the same journey by traveling at the rates of 9 km/hr and 8 km/hr respectively. Find the length of the journey when one takes 20 minutes longer than the other.

Sol: Here, change in speed = 9 – 8 = 1
Product of speed = 9 x 8 = 72
Difference of time = 20 min = $\frac{20}{60}$
Length i.e. distance, d =?

We have,

$$\frac{\text{Product of speed}}{d} = \frac{\text{Difference of speed}}{\text{Difference of time}} \Rightarrow \frac{72}{d} = \frac{1}{\frac{20}{60}}$$

$$\Rightarrow d = 72\times\frac{20}{60} = 24\text{km.}$$

10. Without stoppings, a train travels certain distance with an average speed of 100 km/hr and with stoppages; it covers the same distance with an average speed of 80 km/hr. How many minutes per hour the strain stops?

Sol: Here, s_1 = 100 and s_2 = 60

Stoppage time/hr = $\left(\dfrac{s_1 - s_2}{s_1}\right) = \dfrac{100-80}{100} = \dfrac{1}{5}\text{ hr} = 12\text{min.}$

11. A train 500 m long crosses a pole in 6 seconds. Find the speed of the train in km/hr?

Sol: Speed of the train = $\dfrac{\text{Length of the train}}{\text{time taking in crossing the pole}} = \dfrac{500}{6}\text{ m/s}$

$$= \frac{500}{6} \times \frac{18}{5} = 300 \text{ km/hr} .$$

12. A train 150 m long passes a bridge in 24 seconds moving with a speed of 54 km/hr. Find the length of the bridge.

Sol: Speed of the train = $\frac{\text{Length of the train} + \text{Length of the bridge}}{\text{time taking in crossing the bridge}}$

$$\Rightarrow \frac{5}{18} \times 54 = \frac{150 + \text{Length of the bridge}}{24}$$
$$\Rightarrow 150 + \text{Length of the bridge} = 24 \times 15$$
$$\Rightarrow \text{Length of the bridge} = 360 - 150 = 210 \text{ m.}$$

13. A train 120 m long is running with a speed of 68 km/hr. In what time will it pass a man walking at 4 km/hr in the opposite direction to that of train?

Sol: Here, L_1 = 120, L_2 = 0, s_1 = 68, s_2 = 4

$L_1 + L_2$ = 120 + 0 = 120 m

$s_1 + s_2$ = 68 + 4 = 72 km/hr = $72 \times \frac{5}{18} = 20 \text{ m/sec.}$

Required time = $\left(\frac{L_1 + L_2}{s_1 + s_2}\right) = \frac{120}{20} = 6 \text{ sec.}$

14. Two trains of length 120 m and 80 m running on parallel tracks in the same direction with a speed of 48 km/hr and 50 km/hr respectively. In what time will they pass each other?

Sol: Here, L_1 = 120, L_2 = 80, s_1 = 48, s_2 = 50

$L_1 + L_2$ = 120 + 80 = 200 m

s_1 - s_2 = 50 - 48 = 2 km/hr = $2 \times \frac{5}{18}$ m/sec.

Required time = $\left(\frac{L_1 + L_2}{s_2 - s_1}\right) = \frac{200}{2 \times \frac{5}{18}} = \frac{200 \times 18}{2 \times 5} = 360 \text{ sec.}$

15. Two trains of lengths 213 m and 205 m run on parallel tracks. When running in the same direction the faster train crosses the slower one in $9\frac{1}{2}$ seconds. When running in opposite direction with the same speeds, they pass each other completely in $5\frac{1}{2}$ seconds. Find the speed of each train.

Sol: Here, L_1 = 213, L_2 = 205, T_1 = $\frac{19}{2}$ and T_2 = $\frac{11}{2}$

$L_1 + L_2$ = 213 + 205 = 418

$T_1 + T_2 = \frac{19}{2} + \frac{11}{2} = \frac{30}{2} = 15$

$T_1 - T_2 = \frac{19}{2} - \frac{11}{2} = \frac{8}{2} = 4$

Speed of the faster train = $\frac{L_1 + L_2}{2}\left(\frac{1}{T_1} + \frac{1}{T_2}\right)$

$$= \frac{418}{2}\left(\frac{T_1 + T_2}{T_1 T_2}\right) = \frac{418}{2} \times \frac{15}{\frac{19}{2} \times \frac{11}{2}} = 60 \text{ m/s.}$$

Speed of the slower train = $\frac{L_1 + L_2}{2}\left(\frac{1}{T_1} \quad \frac{1}{T_2}\right)$

$$= \frac{418}{2}\left(\frac{T_1 - T_2}{T_1 T_2}\right) = \frac{418}{2} \times \frac{4}{\frac{19}{2} \times \frac{11}{2}} = 16 \text{ m/s.}$$

16. A train starts from Hyderabad at 9 A.M. with a speed of 50 km/hr and another train starts from there on the same day at 1 P.M. in the same direction with a speed of 70 km/hr. Find at what distance from Hyderabad both the trains will meet and also find the time of their meeting.

Sol: s_1= 50, s_2 = 70, T = time from 9 A.M. to 1 P.M. = 4 hours.

Distance of meeting point from Hyderabad = $\left(\frac{s_1 \times s_2 \times T}{s_2 - s_1}\right)$ km

$$= \left(\frac{50 \times 70 \times 4}{70 - 50}\right) = 700\text{km}.$$

Time of their meeting = $\left(\frac{s_1 \times T}{s_2 - s_1}\right)\text{hr} = \frac{50 \times 4}{20} = 10\text{hr}$ after 1 P.M.

i.e. at 11 P.M. on the same day.

BOATS AND STREAMS

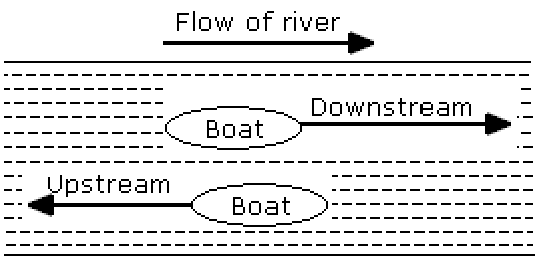
Still Water: If the speed of the water in the river is Zero, then the water is called Still water. (Stationary Water)

Stream: If the water of the river is moving, it is called a stream. (Water in motion)

Upstream: If a boat or a swimmer moves against the stream i.e. in the direction opposite to that of the stream, it is called upstream.

Downstream: If a boat or a swimmer moves with the stream i.e. along the direction of the stream, it is called downstream.

Note: If the speed of a boat or a swimmer is given, it usually means the speed in still water.



Formulae:

1. If the speed of a boat or a swimmer be x km/hr and the speed of the stream or the current be y km/hr, then

a) Speed of the boat or swimmer downstream = $(x + y)$ km/hr.
b) Speed of the boat or swimmer downstream = $(x - y)$ km/hr.

2. a) Speed of the boat or swimmer in still water

$= \frac{1}{2}$ (Downstream speed + Upstream speed)

b) Speed of the stream

$= \frac{1}{2}$ (Downstream speed - Upstream speed)

3. If a man is capable of rowing at the speed of x km/hr in still water, rows the same distance up and down a stream which flows at a rate of y km/hr, then his average speed throughout the journey =

$$\frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}} = \frac{(x+y)(x-y)}{x} \text{ km/hr.}$$

4. A man can row a boat in still water at x km/hr. In a stream flowing at y km/hr, if it takes t hours more in upstream than to go downstream for the same distance, then

The distance = $\frac{(x - y)t}{\ }$ km.

$2y$

a. A man rows a certain distance downstream in t_1 hours and returns the same distance upstream in t_2 hours. If the speed of the stream be x km/hr, then

The speed of the man in still water = $y\left(\frac{t_2+t_1}{t_2-t_1}\right)$ km/hr.

b. A man rows a boat in still water at x km/hr. In a stream flowing at y km/hr if it takes him t hours to row to a place and come back, then

The distance between the two places = $t\left(\frac{x^2-y^2}{2x}\right)$ km.

c. A boat or a swimmer takes n times as long to row upstream so as to row downstream the river. If the speed of boat or swimmer be x km/hr and the speed of stream be y km/hr,

Then $x = y\left(\frac{n+1}{n-1}\right)$.

Solved Examples

1. The speed of a boat in still water is 10 km/hr. If the speed of the stream be 2 km/hr, then find its downstream and upstream speeds.

Sol: Speed of the boat (x) = 10 km/hr.
Speed of the stream (y) = 2 km/hr.
Downstream speed = x + y = 10 + 2 = 12 km/hr.
Upstream speed = x – y = 10 – 2 = 8 km/hr.

2. A boat is rowed down a river 32 km in 4 hours and up a river 15 km in 3 hours. Find the speed of the boat and the river.

Sol: Speed of the boat downstream = $\frac{32}{4} = 8$ km/hr.

Speed of the boat upstream = $\frac{15}{3} = 5$ km/hr.

Speed of the boat = $\frac{1}{2}$ (Downstream speed + Upstream speed)

= $\frac{1}{2}$ (8 + 5) = $\frac{13}{2} = 6.5$ km/hr.

Speed of the river = $\frac{1}{2}$ (Downstream speed - Upstream speed)

= $\frac{1}{2}$ (8 - 5) = $\frac{3}{2} = 1.5$ km/hr.

3. A man rows a boat at a speed of 16 km/hr in still water to a certain distance upstream and back to the starting point in a river which flows at 8 km/hr. Find his average speed for total journey.

Sol: Average speed = $\frac{\text{Upstream} \times \text{Downstream}}{\text{Man's rate in still water}} = \frac{(x+y)(x-y)}{x}$

= $\frac{(16+8)(16-8)}{16} = \frac{24\times 8}{16} = 12$ km/hr.

4. A man can row 4 km/hr in still water. If the river is running at 3 km/hr, it takes 12 hours more in upstream than to go downstream for the same distance. How far is the place?

Sol: The required distance = $\frac{(x^2-y^2)t}{2y} = \frac{(4^2-3^2)\times 12}{2\times 3} = 10$ km.

5. A motorboat covers a certain distance downstream in 8 hours but takes 10 hours to return upstream to the starting point. If the speed of the stream be 14 km/hr, then find the speed of the motor boat in still water.

Sol: Speed of the motorboat in still water = $y\left(\frac{t_2+t_1}{t_2-t_1}\right)$ km/hr

= $14\left(\frac{10+8}{10-8}\right)=126$ km/hr.

6. A man can row 8 km/hr in the still water. If the river is running at 4 km/hr, it takes him 10 hours to row to a place and back. How far is the place?

Sol: The required distance = $t\left(\frac{x^2-y^2}{2x}\right)$ km

= $10\times\left(\frac{8^2-4^2}{2\times 8}\right)=\frac{10\times 48}{16}=30$ km.

7. A man can row at the rate of 16 km/hr in still water. If the time taken to row a certain distance upstream is 7 times as much as to row the same distance downstream, find the speed of the current.

Sol: Speed of the man = $\left(\frac{n+1}{n-1}\right)$ speed of the current

$\Rightarrow$ 16 = $\left(\frac{7+1}{7-1}\right)$ speed of the current

So, speed of the current = 12 km/hr.

Exercise - 10

1. A car moves at a speed of 80km/hr. What is the speed of the car in meters per second?
   1) $12\frac{2}{9}$ 2) $22\frac{2}{9}$
   3) $20\frac{1}{9}$ 4) $21\frac{9}{2}$
2. If a man can cover 12 meters in one second, how many kilo meters can be cover in 3 hours 45 minutes?
   1) 168 km 2) 162 km
   3) 150 km 4) 156 km
3. If a man running at 15 kmph. Crosses a bridge in 5 minutes, then the length of the bridge is ______________
   1) 1230 m 2) 1240 m
   3) 1250 m 4) 1220 m
4. Walking at $\frac{3}{4}^{th}$ of his usual speed a man is late by 2 hours 30 minutes. The usual time would have been
   1) $7\frac{1}{2}$ hrs 2) $3\frac{1}{2}$ hrs
   3) $3\frac{1}{4}$ hrs 4) $\frac{7}{8}$ hrs
5. In a 1 km race, A beats B by 100 m and C by 150 m. In a 2700 m race, by how many meters does B beat C?
   1) 100 m 2) 120 m
   3) 150 m 4) 180 m
6. Traveling at a speed of 8 kmph a student reaches school from his house 10 minutes early. If he travels at 6 kmph, he is late by 20 minutes. Find the distance between the school and the house.
   1) 12 km 2) 1 km
   3) 10 km 4) 13 km
7. A man takes 5 hours 45 minutes in walking to a certain place and riding back. He could have gained 2 hours by riding both ways. The time he would take to walk both ways is __________
   1) 12 hrs 2) 11 hrs 45minutes
   3) 7 hrs 45 minutes 4) 3 hrs

8. The ratio between rates of walking of two persons is 3:4. If the time taken by 2^{nd} person to cover a certain distance is 36 minutes, then the time taken by the first person to cover the same distance is ____________
   1) 36 minutes 2) 48 minutes
   3) 27 minutes 4) none

9. If the speed of a vehicle changes in the ratio a : b, then the ratio of times taken is _____________
   1) a : b 2) $\frac{a}{b-1}$
   3) b : a 4) $\frac{b}{a-1}$

10. A car driver makes his journey by the speed of 75km/hr and returns to initial place with 50 km/hr. Then his average speed of journey is ____________
    1) 30 km/hr 2) 40 km/hr
    3) 50 km/hr 4) 60 km/hr

11. A vehicle travels 715 km at a uniform speed. If the speed of the car is 10 kmph more, it takes 2 hours less to cover the same distance. The original speed was ________________
    1) 45 kmph 2) 65 kmph
    3) 55 kmph 4) 75 kmph

12. Two persons P and Q run at 8 kmph and 12 kmph on a circular track of length 6 km in the same direction starting at same time from same place. After how many hours will they meet each other any where on the track?
    1) 1.5 hours 2) 2 hours
    3) 2.5 hours 4) 3.5 hours

13. A car driver driving at a speed of 68kmph locates a truck 40 meters ahead of him. After 10 seconds, the truck is 60 meters behind. The speed of truck is _____________
    1) 30 km/hr 2) 32 km/hr
    3) 23 km/hr 4) 3 km/hr

14. Rajan is traveling on his cycle and has calculated to reach a point at 2 p.m. if he travels at 10 kmph. He would reach there by 12 noon if he travels at 15 kmph. At what speed must he travel to reach the same place at 1.p.m?
    1) 12 kmph 2) 14 kmph
    3) 15 kmph 4) 13 kmph

15. Two persons start running simultaneously around a circular track of length 300m from the same point at speeds 15 and 25km/hr. When will they meet first time on the track, when move in opposite direction?
    1) 21 sec 2) 22 sec
    3) 27 sec 4) 24 sec

16. A robber steals a Maruthi car at 2.30 pm and drives at 60 kmph. The theft is discovered at 3 p.m. and the owner sits in Police jeep running at 75 kmph. When will he catch the thief?
    1) 5.30 pm 2) 5.15 pm
    3) 5 pm 4) 5.45 pm

17. Two planes move along a circle of circumference 1.2 km with constant speeds. When they move in different directions they meet every 15 sec and then they move in the same direction one plane over takes the other every 60 sec. The speed of slower plane is
    1) 0.04 km/s 2) 0.03 km/s
    3) 0.05 km/s 4) 0.02 km/s

18. A 150 m long train crosses a man walking at a speed of 6 kmph in his opposite direction in 6 sec. The train (in kmph) is:
    1) 66 2) 84
    3) 96 4) 106

19. A train of length 150 m takes 10 sec to pass over another train 100 m long coming from the opposite direction. If the speed of the train is 30 kmph. Then the speed of the second train in kmph is __________
    1) 54 2) 60
    3) 72 4) 36

20. If a train 110m long passes a signal pole in 3 sec. Then the time taken by it to cross a railway platform 165m long is :
    1) 3secs 2) 4secs
    3) 7.5secs 4) 5secs

21. An Engine of 10 m length travels at 60 kmph. How long does it take to cross another train 170 m long, running at 54 kmph in the same direction?
    1) 16 sec 2) 16.8 sec
    3) 108 sec 4) none

22. Two trains starting at the same time from two stations 200 km apart and going in opposite directions cross each other at a distance of 110 km from one of the stations. What is the ratio of

their speeds?

1) 9:11 2) 11:9
3) 10:9 4) 9:10

23. A train M leaves Mumbai at 5am. And reaches Delhi at 9am. Another train leaves Delhi at 7am. And reaches Mumbai at 11.00am. At what time do the two trains across each other?
1) 8 a.m. 2) 9 a.m.
3) 7 a.m. 4) 6 a.m.

24. Train P leaves Hyderabad at 6.00am. And reaches Vijayawada at 10.00am. Train Q leaves Vijayawada at 7.00am. And reaches Hyderabad at 1.00pm. At what time do the trains meet?
1) 8.48 a.m. 2) 8.12 a.m.
3) 8.42 a.m. 4) 9.00 a.m.

25. A train running at 52kmph takes 36 seconds to pass a platform. Next it takes 24 seconds to pass a man walking at 10 kmph in the same direction. Find the length of the train and that of the platform?
1) 800 m; 440 m 2) 280 m; 440 m
3) 280 m; 240 m 4) 420 m; 300 m

26. Two trains running in the same direction at 40 kmph and 22 kmph completely pass one another in 60 seconds. If the length of the first train is 125 meters, then the length of second train is?
1) 125 m 2) 128 m
3) 175 m 4) 900 m

27. Two trains 220 meters and 380 meters in length respectively are running in opposite direction. One at the rate of 35 kmph and other at 25 kmph. In what time they will cross each other?
1) 36 seconds 2) 30 seconds
3) 60 seconds 4) None

28. A man misses a train by 40 minutes if he travels at 30 kmph. If he travels at 40 kmph, then also he misses the train by 10minutes. What is the minimum speed required to catch the train on time?
1) 44 kmph 2) 45 kmph
3) 48 kmph 4) 49 kmph

29. A boat traveled from A to B and back to A from B in 5 hours. If the speed of boat in still water and the speed of stream be 7.5 kmph and 1.5 kmph, then what is the distance between A and B?
1) 80 km 2) 45 km
3) 18 km 4) 19 km

30. A man can row downstream at 18 kmph and upstream at 10 kmph. Find the speed of the man in still water and the speed of stream (in kmph)
1) 13; 3 2) 15; 3
3) 12; 6 4) 14; 4

31. A man can row at 9 kmph in still water. He takes 4 ½ hours to row from P to Q and back. What is the distance between P and Q if the speed of the stream is 1 kmph?
1) 32 km 2) 28 km
3) 20km 4) 24 km

32. A man can row 30 km downstream in 3 hours 45 minutes, and 11 km upstream in 2 hours 12 minutes. What is the speed of the man in still water and speed of stream (in kmph)?
1) 6; 2 2) 6.8; 1.8
3) 6.5; 1.5 4) 7; 3

33. A man rows 22 km upstream in 4 hours and 45 km downstream in 6 hours. In 10 hours how much more distance can he row downstream than the distance he can row upstream?
1) 24 km 2) 22 km
3) 20 km 4) 18 km

34. A person can row 10 km in 1 hour in still water. If the speed of the water current is 2 kmph and it takes two hours for him to go to a certain place and back. Find the distance he traveled in upstream?
1) 9 ½ km 2) 9.6 km
3) 48 km 4) 5 km

35. A person can row $\frac{3}{5}$ of a km in upstream in 10 minutes and return in 6 minutes. Find the speed of man in still water?
1) 4.4 kmph 2) 4.5 kmph
3) 4.8 kmph 4) 4.9 kmph

36. A boat can travel 10 kmph in still water. It traveled 91 km downstream and then returned, taking altogether 20 hours. Find speed of the stream?

1) 4 kmph
2) 5 kmph
3) 8 kmph
4) 3 kmph

37. The time taken for a boat to cover certain distance in upstream is equal to the time taken by the boat to cover three times the distance in downstream. If the speed of current is 5 kmph, what is the speed of boat in still water?
1) 14 kmph
2) 15 kmph
3) 10 kmph
4) 19 kmph

38. The time taken by a person to row upstream is twice the time taken by him to row the same distance downstream. If the speed of the boat in still water is 42 kmph, find the speed of current?
1) 14 kmph
2) 32 kmph
3) 12 kmph
4) 8 kmph

39. A man rows his boat to a certain place covering a distance of 72 km and back again in 15 hours. He finds that he takes same time to row 3 km in downstream as much he takes for 2 km in upstream. Find the speed of the stream?
1) 4 kmph
2) 3 kmph
3) 1 kmph
4) 2 kmph

40. A man can row 6 km/hr in still water. If the speed of stream is 2km/hr, it takes him 3 hours to row to a place and back. How far is the place?
1) 16 km
2) 10 km
3) 12 km
4) 8 km

ALLIGATION OR MIXTURE

Alligation: It means linking.

a) It is a rule to find the ratio in which two or more ingredients at their respective prices should be mixed to give a mixture at a given price.

b) It is a rule to find the average price of a mixture when the prices of two or more ingredients which may be mixed together and the proportion in which they are mixed are given.

Mean Price: The cost price of a unit quantity of mixture is called the mean price.

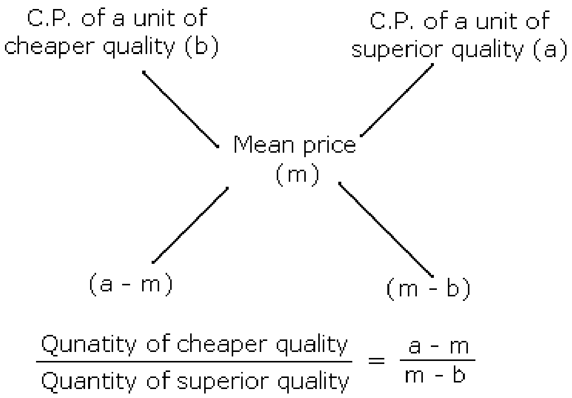
Alligation Rule:

1. Let us suppose Rs.a per unit be the price of the first ingredient (superior quality) is mixed with another ingredient (cheaper quality) of price Rs.b per unit to form a mixture whose mean price is Rs.m per unit, then the two ingredients must be mixed in the ratio:

$$\frac{\text{Quantity of cheaper}}{\text{Quantity of superior}} = \frac{\text{C.P. of superior - Mean price}}{\text{Mean price - C.P.of cheaper}} = \frac{a-m}{m-b}$$

2. This means, the two ingredients are to be mixed in the inverse ratio of the differences of their prices and the mean price.

Note: This can be remembered easily through the diagram below:



Formulae:

1. A vessel contains a litres of wine. From it b litres are withdrawn. The vessel is then filled with water. Next b litres of the mixture are withdrawn and again the vessel is filled with water. If this process is repeated n times then

$$\frac{\text{Wine left in the vessel after nth operation}}{\text{Original quantity of wine in the vessel}} = \left(\frac{a-b}{}\right)^n$$

a

2. There are n vessels of equal size filled with mixtures of liquids A and B in the ratio $a_1 : b_1$, $a_2 : b_2$,………,$a_n : b_n$, respectively. If the contents of all the vessels are poured into a single large vessel, then

$$\frac{\text{Quantity of liquid A}}{\text{Quantity of liquid B}} = \frac{\left(\frac{a_1}{a_1+b_1}+\frac{a_2}{a_2+b_2}+\ldots\ldots\ldots\ldots\ldots+\frac{a_n}{a_n+b_n}\right)}{\left(\frac{b_1}{a_1+b_1}+\frac{b_2}{a_2+b_2}+\ldots\ldots\ldots\ldots\ldots+\frac{b_n}{a_n+b_n}\right)}$$

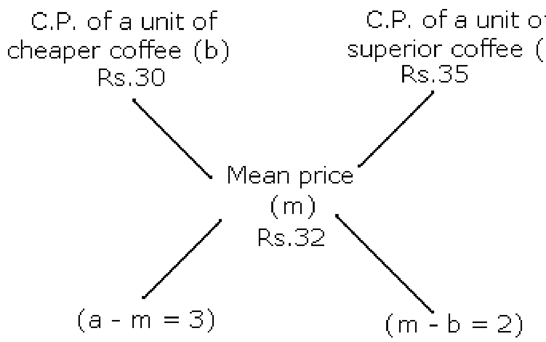
3. There are n vessels of sizes c_1, c_2, …….. , c_n filled with mixtures of liquids A and B in the ratio $a_1 : b_1$, $a_2 : b_2$,………,$a_n : b_n$, respectively. If the contents of all the vessels are poured into a single large vessel, then

$$\frac{\text{Quantity of liquid A}}{\text{Quantity of liquid B}} = \frac{\left(\frac{a_1c_1}{a_1+b_1}+\frac{a_2c_2}{a_2+b_2}+\ldots\ldots\ldots\ldots\ldots+\frac{a_nc_n}{a_n+b_n}\right)}{\left(\frac{b_1c_1}{a_1+b_1}+\frac{b_2c_2}{a_2+b_2}+\ldots\ldots\ldots\ldots\ldots+\frac{b_nc_n}{a_n+b_n}\right)}$$

Solved Examples

1. In what ratio the two varieties of coffee one costing Rs.30 per kg and the other Rs.35 per kg should be blended to produce a blended variety of coffee worth Rs.32 per kg. How much should be the quantity of second variety of coffee, if the first variety is 72 kg.

Sol:



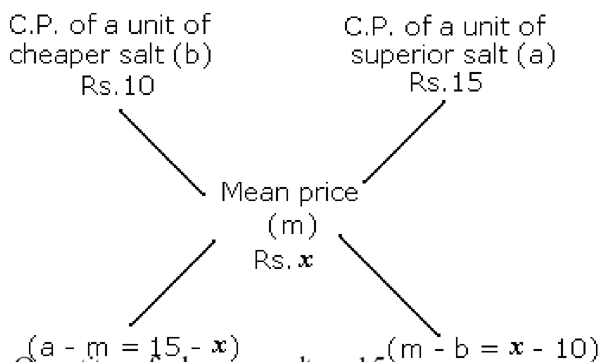
The required ratio of the two varieties of coffee is 3 : 2. i.e.

$$\frac{\text{Quantity of cheaper quality}}{\text{Quantity of superior quality}} = \frac{3}{2}$$

So, Quantity of superior coffee = $72 \times \frac{2}{3} = 48$ kg.

2. Salt at Rs.10 per kg is mixed with salt at Rs.15 per kg in the ratio 3 : 4. Find the price per kg of mixture.

Sol: Let the mean price of the mixture be Rs.x.



$$\frac{\text{Quantity of cheaper salt}}{\text{Quantity of superior salt}} = \frac{15-x}{x-10} \Rightarrow \frac{3}{4} = \frac{15-x}{x-10} \Rightarrow 3x-30 = 60-4x \Rightarrow 7x = 90 \Rightarrow x = \frac{90}{7} = Rs.12\frac{6}{7}.$$

3. A vessel contains 100 litres of wine. 20 litres of wine was taken out and replaced by water. Then, 20 litres of mixture was withdrawn and again replaced by water. The operation was repeated for fourth time. How much wine is now left in the vessel?

Sol: Amount of wine left in the vessel = $\left(1-\frac{20}{100}\right)^4 \times 100 = \left(\frac{4}{5}\right)^4 \times 100 = 40.96 litres.$

4. Two equal glasses are filled with mixture of milk and water. The proportion of milk and water in the first glass is 4 : 3 and in the second glass is 5 : 2. The contents of the two glasses are emptied into a single vessel. What is the proportion of milk and water in it?

Sol: $\frac{\text{Quantity of milk}}{\text{Quantity of water}} = \frac{\frac{4}{4+3}+\frac{5}{5+2}}{\frac{3}{4+3}+\frac{2}{5+2}} = \frac{9}{5} = 9:5.$

5. Three glasses of sizes 1 litres, 2 litres and 3 litres contain mixture of milk and water in the ratio 2: 3, 3 : 7 and 4 : 11 respectively. The contents of all the three glasses are poured into a single vessel. Find the ratio of milk to water in the resulting mixture.

Sol: $\frac{\text{Quantity of milk}}{\text{Quantity of water}} = \frac{\left(\frac{2\times1}{2+3}+\frac{3\times2}{3+7}+\frac{4\times3}{4+11}\right)}{\left(\frac{3\times1}{2+3}+\frac{7\times2}{3+7}+\frac{11\times3}{4+11}\right)} = \frac{\left(\frac{2}{5}+\frac{6}{10}+\frac{12}{15}\right)}{\left(\frac{3}{5}+\frac{14}{10}+\frac{33}{15}\right)} = \frac{\frac{12+18+24}{30}}{\frac{18+42+66}{30}} = \frac{54}{126}$

= 54 : 126 or 27 : 63.

Exercise - 11

1. How many kg of coffee worth Rs.25 per kg must be blended with 30 kg of coffee worth Rs.30 per kg so that on selling the blended variety at Rs.30 per kg there should be a gain of 10%?
1) 36 kg 2) 40 kg
3) 32 kg 4) 42 kg

2. How much water be added to 14 litres of alcohol worth Rs.5.40 a litre so that the value of the mixture may be Rs.4.20 a litre?
1) 7 litres 2) 6 litres
3) 5 litres 4) 4 litres

3. In what ratio two varieties of tea, one costing Rs.25 per kg and the other costing rs.30 per kg should be blended to produce blended variety of tea worth Rs.28 per kg?
1) 3 : 4 2) 4 : 3
3) 2 : 3 4) 3 : 5

4. In an examination out of 80 students 85% of the girls and 70% of the boys passed. How many boys appeared in the examination if total pass percentage was 75%?
1) 370 2) 340
3) 320 4) 360

5. In what proportion must tea worth 75 paise per packet be mixed with tea worth Rs.5.50 per packet so that the mixture may cost Rs.4.50 per packet?
1) 3 : 11 2) 4 : 15
3) 15 : 11 4) 4 : 5

6. How many kg of sugar costing Rs.5.50 per kg must be mixed with 60 kg of sugar costing Rs.4.80 per kg so that the mixture is worth Rs.5.25 per kg?
1) 90 kg 2) 95 kg
3) 108 kg 4) 106 kg

7. In what ratio must water be added to spirit to gain 10% by selling it at the cost price?
1) 1 : 11 2) 1 : 5
3) 1 : 10 4) 1 : 9

8. 300 gm of salt solution has 40% salt in it. How much salt should be added to make it 50% in the solution?

1) 40 gm
2) 60 gm
3) 70 gm
4) 80 gm

9. A man buys two cows for Rs.1350 and sells one so as to lose 6% and the other so as to gain 7.5% and on the whole he neither gains nor loses. What does each cow cost?
1) 36, 39
2) 42, 45
3) 39, 36
4) 45, 48

10. There are 65 students in a class. Rs.39 is distributed among them so that each boy gets 80 p and girl gets 30 p. Find the number of boys and girls in that class.
1) 43, 40
2) 36, 33
3) 39, 26
4) 45, 42

11. A trader has 50 kg of sugar, a part of which he sells at 10% profit and the rest at 5% loss. He gains 7% on the whole. What is the quantity sold at 10% gain and 5% loss?

1) 40 kg, 10 kg
2) 10 kg, 35 kg
3) 25 kg, 15 kg
4) 30 kg, 20 kg

12. 729 ml of a mixture contains milk and water in the ratio 7 : 2. How much more water is to be added to get a new mixture containing milk and water in the ratio 7 : 3?
1) 600 ml
2) 710 ml
3) 520 ml
4) None

13. A trader has 50 kg of pulses, part of which he sells at 8% profit and the rest at 18% profit. He gains 14% on the whole. What is the quantity sold at 18% profit?
1) 30 kg
2) 25 kg
3) 20 kg
4) 40 kg

14. A trader has 50 kg of rice, a part of which he sells at 10% profit and the rest at 5% loss. He gains 7% on the whole. What is the quantity sold at 10% gain and 5% loss?
1) 30 kg, 10 kg
2) 40 kg, 15 kg
3) 35 kg, 40 kg
4) 40 kg, 10 kg

15. Meghana's expenditure and savings are in the ratio 3 : 2. Her income increases by 10%. Her expenditure also increases by 12%. By how many % does her saving increase?
1) 7%
2) 10%
3) 9%
4) 13%

16. 40 litres of mixture of milk and water contains 10% of water. The water to be added to make water content 20% in the new mixture is ….
1) 6 litres
2) 6.5 litres
3) 5.5 litres
4) 5 litres

17. Two types of steel are available. Type A contains 5% of nickel and type B contains 40% of nickel. How much of each type be mixed to obtain 140 tonnes of steel containing 30% of nickel?
1) 40, 1000 tonnes
2) 50, 125 tonnes
3) 30, 75 tonnes
4) 60, 180 tonnes

18. In what ratio water should be mixed with a liquid at Rs.12 per litre so that by selling the mixture at Rs.13.75 per litre, the seller gains 25%?
1) 1 : 11
2) 1 : 15
3) 1 : 17
4) 1 : 13

19. A tea producer mixes two categories of tea in the ratio 5 : 3. The cost price of one category is Rs.27 per kg and that of other is Rs.30 per kg. The mixture is sold at Rs.30.25 a kg. Find his gain per cent.
1) $8\frac{5}{3}$
2) $7\frac{5}{9}$
3) $9\frac{5}{9}$
4) $11\frac{5}{9}$

20. In a mixture of 60 litres, the ratio of milk and water is 2 : 1. If the ratio of the milk and water is to be 1 : 2. If the ratio of the milk and water is to be 1 : 2, then the amount of water to be further added to
1) 20 litres
2) 30 litres
3) 40 litres
4) 60 litres

CLOCKS

Minute Spaces: The circumference of a dial of a clock or watch is divided into 60 equal parts. These parts are called minute spaces.

Hands: The clock has two hands. 1) Hour hand 2) Minute hand

Hour hand: The hour hand or short hand indicates time in hours.
In an hour, the hour hand covers 5 minute spaces.

Minute hand: The minute hand or long hand indicates time in minutes.
In an hour, the minute hand covers 60 minute spaces.

Note:

- In every hour, both the hands coincide once.
- In every hour, the hands are straight (points in opposite directions) once. In this position, the hands are 30 minutes apart.
- In every hour, the hands are twice at right angles. In this position, the hands are 15 minute spaces apart.
- The minute hand moves through 6° in each minute whereas the hour hand moves through $\frac{1}{2}$ in each minute. Thus, in one minute, the minute hand gains $5\frac{1}{2}^\circ$ than the hour hand.
- When the hands coincide, the angle between them is 0°.
- When the hands point in opposite directions, the angle between them is 180°.
- The hands are in the same straight line, when they are coincident or opposite to each other. So, the angle between the two hands is 0° or 180°.
- The minute hand moves 12 times as fast as the hour hand.

Formulae:

1. The two hands of the clock will be together between H and $(H + 1)$ O' clock at $\left(\frac{60H}{11}\right)$ O' clock.

2. The two hands of the clock will be at right angles between H and $(H + 1)$ O' clock at $(5H \pm 15)\frac{12}{11}$ minutes past H O' clock.

3. The two hands of the clock will be in the same straight line but not together between H and $(H + 1)$ O' clock at

$(5H - 30)\frac{12}{11}$ minutes past H when $H > 6$,

$(5H + 30)\frac{12}{11}$ minutes past H when $H < 6$

4. Between H and $(H + 1)$ O' clock, the two hands of a clock are M minutes apart at $(5H \pm M)\frac{12}{11}$ minutes past H O' clock.

5. Angle between hands of a clock:

a) When the minute hand is behind the hour hand, the angle between the two hands at M minutes past H O' clock = $30\left(H - \frac{M}{5}\right) + \frac{M}{2}$ degrees.

b) When the minute hand is ahead of the hour hand, the angle between the two hands, at M minutes past H O' clock = $30\left(\frac{M}{5} - H\right) - \frac{M}{2}$ degrees.

6. The minute hand of a clock overtakes the hour hand at intervals of M minutes of correct time. The clock gains or loses in a day by $\left(\frac{720}{11} - M\right)\left(\frac{60 \times 24}{M}\right)$ minutes.

Solved Examples

1. At what time between 6 and 7 O' clock are hands of a clock together?

Sol: Here, $H = 6$

$$\therefore \left(\frac{60H}{11}\right) = \frac{60 \times 6}{11} = \frac{360}{11} = 32\frac{8}{11}$$

$\therefore$ Hands of a clock are together at $32\frac{8}{11}$ minutes past 6 O' clock.

2. At what time between 7 and 8 O' clock will the hands of a clock be at right angle?

Sol: Here, $H = 7$

$$\therefore (5H \pm 15)\frac{12}{11} = (5 \times 7 \pm 15)\frac{12}{11} = 54\frac{6}{11} \text{ and } 21\frac{9}{11}$$

11 11

∴ Hands of a clock are at right angle at $54\frac{6}{11}$ minutes past 7 and $21\frac{9}{11}$ minutes past 7.

3. Find at what time between 3 and 4 O' clock will the hands of a clock be in the same straight line but not together.

Sol: Here, H = 3 < 6

∴ $(5H + 30)\frac{12}{11} = (5 \times 3 + 30)\frac{12}{11} = \frac{540}{11} = 49\frac{1}{11}$.

∴ The hands will be in the same straight line but not together at $49\frac{1}{11}$ minutes past 3 O' clock.

4. Find the time between 5 and 6 O' clock when the two hands of a clock are 5 minutes apart.

Sol: Here, H = 5 and M = 5

∴ $(5H \pm M)\frac{12}{11} = (5 \times 5 \pm 5)\frac{12}{11} = 32\frac{8}{11}$ and $21\frac{9}{11}$

∴ The hands will be 5 minutes apart at $32\frac{8}{11}$ past and $21\frac{9}{11}$ past 5 O' clock.

5. Find the angle between two hands of a clock at 25 minutes past 8 O' clock.

Sol: Here, H = 8 and M = 25

∴ The required angle = $30\left(H - \frac{M}{5}\right) + \frac{M}{2}$ degrees $= 102.5^{o}$

$= 30\left(8 - \frac{25}{5}\right) + \frac{5}{2}$ degrees

$= \frac{205}{2}$.

Exercise - 12

1. What is the angle between the hands of the clock at 2.45?

1) $180\frac{1}{2}^{o}$ 2) $182\frac{1}{2}^{o}$

3) $172\frac{1}{2}^{o}$ 4) $181\frac{1}{2}^{o}$

2. At 9^{o} clock find the angle between the hands of the clock.

1) $270\frac{1}{2}^{o}$ 2) $250\frac{1}{2}^{o}$

3) $150\frac{1}{2}^{o}$ 4) $220\frac{1}{2}^{o}$

3. At what time between 6 O' clock and 7 O' clock the hands of the clock will coincide?

1) $30\frac{8}{11}$ min 2) $32\frac{8}{11}$ min

3) $20\frac{8}{11}$ min 4) $25\frac{8}{11}$ min

4. At what time between 6 O' clock and 4 O' clock the hands of the clock will be at right angles?

1) $30\frac{8}{11}$ min 2) $10\frac{8}{11}$ min

3) $32\frac{8}{11}$ min 4) $34\frac{8}{11}$ min

5. At what time between 1 O' clock and 2 O' clock the hands of the clock will be in opposite direction?

1) $30\frac{8}{11}$ min 2) $20\frac{8}{11}$ min

3) $10\frac{8}{11}$ min | 4) $32\frac{4}{11}$ min

6. At what time between 5 O' clock and 6 O' clock the hands of the clock will be at 180^o ?
1) 60 min | 2) 50 min
3) 40 min | 4) 30 min

7. A clock which gains 6 minutes every three hours is set at 1.00 P.M. on a certain day. Find the time shown by the watch on next day at 13:50 hours.
1) 15 hrs 21 min 40 sec | 2) 15 hrs 11 min 40 sec
3) 15 hrs 41 min 40 sec | 4) 15 hrs 31 min 40 sec

8. A clock which loses 5 minutes in every hour is set at 10.30 A.M. on a certain day. Next day at 18.00 hours what is the time shown by this watch?
1) 15 hrs 12 min 30 sec | 2) 15 hrs 22 min 30 sec
3) 15 hrs 31 min 40 sec | 4) 15 hrs 41 min 30 sec

9. A clock, which loses 40 seconds every four minutes, is set at 18.00 hours on a certain day. What is the time shown by this watch if the current time is 4.00 P.M.?
1) 12:10 P.M. | 2) 11:10 P.M.
3) 10:10 P.M. | 4) 9:10 P.M.

10. A clock, which gains 6 minutes in every three hours, is set at 6 P.M. on a certain day. If on next day, the time shown by this watch is 11 P.M. what is the correct time?
1) 8 hrs 2 min (PM) | 2) 8 hrs 4 min (PM)
3) 8 hrs 8 min (PM) | 4) 8 hrs 6 min (PM)

11. A clock, which loses 5 minutes in every two hours, is set at 9.00 A.M. on a certain day. Next day if the time shown by this watch is 11 P.M. what is the correct time?
1) $10\frac{15}{23}$ min | 2) $11\frac{15}{23}$ min
3) $9\frac{15}{23}$ min | 4) $12\frac{15}{23}$ min

12. How many times in a day the hands of a clock are straight?
1) 20 | 2) 22
3) 19 | 4) 18

13. How many times do the hands of a clock point towards each other in a day?
1) 18 | 2) 19
3) 22 | 4) 20

14. A clock which gains 5 minutes in every two hours is set at 12.00 P.M. on a certain day. Find the time shown by the watch on the next day 11 A.M.
1) 12 hrs 47 min 30 sec | 2) 12 hrs 45 min 30 sec
3) 12 hrs 20 min 30 sec | 4) 12 hrs 15 min 30 sec

15. A clock which loses 10 seconds in every minute is set at 2.00 P.M. on a certain day. Find the time shown by the watch on the next day 8 P.M.
1) 1 P.M. | 2) 2 P.M.
3) 3 P.M. | 4) 4 P.M.

16. A clock which loses 50 seconds every two minutes is set at 6.00 P.M. on a certain day. What is the time shown by this watch if the current time is 3.00 P.M.?
1) 4 P.M. | 2) 9 P.M.
3) 5 P.M. | 4) 6 P.M.

17. A clock which gains 5 minutes in every two hours is set at 12 noon on a certain day. If on the next day, the time shown by this watch is 1 P.M. then find the correct time.
1) 10 P.M. | 2) 12 P.M.
3) 11 P.M. | 4) 8 P.M.

18. A clock, which loses 3 minutes in every hour, is set at 10.00 A.M. on a certain day. Next day if the time shown by this watch is 2.30 P.M. What is the correct time?
1) $19\frac{1}{29}$ min | 2) $19\frac{3}{29}$ min
3) $19\frac{4}{29}$ min | 4) $19\frac{2}{29}$ min

19. How many times do the hands of a clock coincide in a day?
1) 22 | 2) 23
3) 24 | 4) 48

20. How many times are the hands of a clock at right angles in a day?

1) 22
2) 48
3) 34
4) 46

21. **If a** clock takes 22 seconds to strike 12, how much time will it take to strike 6?
1) 6 sec
2) 10 sec
3) 8 sec
4) None

22. At what angle the hands of a clock are inclined when the time is 15 minutes past five?
1) $67\frac{1}{2}^{o}$
2) 68^{o}
3) 70^{o}
4) None

23. At what time between 3 and 4 O' clock are the hands of a clock together?
1) 18 min past 3
2) $12\frac{6}{4}$ min past 8
3) $16\frac{4}{11}$ min past 3
4) None

24. At what time between 8 and 9 O' clock are the hands of a clock be in the straight line but not together?
1) $10\frac{5}{11}$ min past 3
2) $10\frac{10}{11}$ min past 8
3) $8\frac{10}{11}$ min past 8
4) None

25. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of correct time. How much in a day does the clock gain or lose?
1) $10\frac{10}{143}$ min
2) $10\frac{10}{11}$ min
3) $10\frac{10}{43}$ min
4) None

26. A watch gains uniformly, is 5 min slow at 8 O' clock in the morning on a Monday, and is 5 min.48 seconds fast at 8 P.M. on following Monday. When was it correct?
1) 3 days 11 hours 20 min2) 8 min
3) 6 days 10 hours
4) None

27. A clock is set right at 8 A.M. The clock gains 10 minutes in 24 hours. What will be the true time when the clock indicates 1 P.M. on the following day?
1) 28 hours
2) 28 hours 48 min
3) 30 hours
4) None

28. At what time between 7 A.M. and 7.30 A.M. will the two hands of a clock be at right angle to each other?
1) $21\frac{9}{11}$ min past 7
2) $5\frac{5}{11}$ min past 7
3) $21\frac{10}{11}$ min past 7
4) None

29. At what time between 9 O' clock and 10 O' clock will the two hands of a clock be in a straight line but in opposite directions?
1) $16\frac{4}{11}$ min past 9
2) $5\frac{5}{11}$ min past 9
3) $10\frac{11}{10}$ min past 9
4) None

30. A watch which gains uniformly is 6 minutes slow at 4 P.M. on a Sunday and $10\frac{2}{3}$ minutes fast on the following Sunday at 8 A.M. During this period when (Day and Time) was the watch correct?
1) 1.30 A.M. , Tuesday
2) 1.36 P.M., Thursday
3) 1.36 A.M. , Wednesday
4) None

CALENDAR

- Here you mainly deal in finding the day of the week on a particular given date.
- The process of finding this depends on the number of odd days.
- Odd days are quite different from the odd numbers.

Odd Days: The days more than the complete number of weeks in a given period are called odd days.

Ordinary Year: An year that has 365 days is called Ordinary Year.

Leap Year: The year which is exactly divisible by 4 (except century) is called a leap year.

Eg: 1968, 1972, 1984, 1988 and so on are the examples of Leap Years.
1986, 1990, 1994, 1998, and so on are the examples of not leap years.

Note: All Centuries are leap years.

Important Points:

1. An ordinary year has 365 days = 52 weeks and 1 odd day.
2. A leap year has 366 days = 52 weeks and 2 odd days.
3. Century = 76 Ordinary years + 24 Leap years.
4. Century contain 5 odd days.
5. 200 years contain 3 odd days.
6. 300 years contain 1 odd day.
7. 400 years contain 0 odd days.
8. Last day of a century cannot be Tuesday, Thursday or Saturday.
9. First day of a century must be Monday, Tuesday, Thursday or Saturday.

Explanation:

1) 100 years = 76 ordinary years + 24 leap years
= 76 odd days + 24 x 2 odd days
= 124 odd days = 17 weeks + 5 days
∴ 100 years contain 5 odd days.

2) No. of odd days in first century = 5
∴ Last day of first century is Friday.

3) No. of odd days in two centuries = 3
∴ Wednesday is the last day.

4) No. of odd days in three centuries = 1
∴ Monday is the last day.

5) No. of odd days in four centuries = 0

∴ Sunday is the last day.

- Since the order is continually kept in successive cycles, the last day of a century cannot be Tuesday, Thursday or Saturday.
- So, the last day of a century should be Sunday, Monday, Wednesday or Friday.
- Therefore, the first day of a century must be Monday, Tuesday, Thursday or Saturday.

Working Rules:

1) Working rule to find the day of the week on a particular date when reference day is given:

Step 1: Find the net number of odd days for the period between the reference date and the given date (exclude the reference day but count the given date for counting the number of net odd days).

Step 2: The day of the week on the particular date is equal to the number of net odd days ahead of the reference day (if the reference day was before this date) but behind the reference day (if this date was behind the reference day).

2) Working rule to find the day of the week on a particular date when no reference day is given

Step 1: Count the net number of odd days on the given date

Step 2: Write:

For 0 odd days – Sunday
For 1 odd day – Monday
For 2 odd days – Tuesday
. . . .
. . . .
. . . .
For 6 odd days - Saturday

Solved Examples

1. If 11^{th} January 1997 was a Sunday then what day of the week was on 10^{th} January 2000?

Sol: Total number of days between 11^{th} January 1997 and 10^{th} January 2000
= (365 – 11) in 1997 + 365 in 1998 + 365 in 1999 + 10 days in 2000
= (50 weeks + 4 odd days) + (52 weeks + 1 odd day) +
(52 weeks + 1 odd day) + (1 week + 3 odd days)

Total number of odd days = 4 + 1 + 1 + 3 = 9 days = 1 week + 2 days

Hence, 10^{th} January, 2000 would be 2 days ahead of Sunday i.e. it was on Tuesday.

2. What day of the week was on 10^{th} June 2008?

Sol:
10^{th} June 2008 = 2007 years + First 5 months up to May 2008 + 10 days of June

2000 years have 0 odd days.
Remaining 7 years has 1 leap year and 6 ordinary years $\Rightarrow$ 2 + 6 = 8 odd days

So, 2007 years have 8 odd days.

No. of odd days from 1^{st} January 2008 to 31^{st} May 2008 = 3+1+3+2+3 = 12

10 days of June has 3 odd days.

Total number of odd days = 8+12+3 = 23

23 odd days = 3 weeks + 2 odd days.

Hence, 10^{th} June, 2008 was Tuesday.

Exercise - 13

1. Find the number of odd days in 200 days.
1) 2 2) 4
3) 5 4) 6

2. Find the number of odd days in 425 days.

1) 3 2) 4
3) 5 4) 6

3. Find the number of odd days in 49 years.
1) 2 2) 4
3) 5 4) 6

4. What day of the week on 26^{th} Jan 1950?
1) Monday 2) Tuesday
3) Wednesday 4) Thursday

5. Gandhiji was born on 2^{nd} October, 1869. What day was it of the week?
1) Thursday 2) Friday
3) Saturday 4) Sunday

6. What day of the week on 2^{nd} June, 1988?
1) Thursday 2) Friday
3) Sunday 4) Saturday

7. What day of the week on 15^{th} August, 1947?
1) Thursday 2) Friday
3) Sunday 4) Saturday

8. What day of the week on 31^{st} October, 1984?
1) Thursday 2) Friday
3) Sunday 4) Saturday

9. What day of the week on 14^{th} March, 1993?
1) Thursday 2) Friday
3) Sunday 4) Saturday

10. What day of the week on 14^{th} November, 1889?
1) Monday 2) Wednesday
3) Thursday 4) Saturday

11. Monday falls on 4th April 1988. What was the day on 3rd November, 1987?
1) Tuesday 2) Monday
3) Saturday 4) Sunday

12. First January, 1981 was Sunday. What day of the week was 1st Jan 1980?
1) Tuesday 2) Monday
3) Saturday 4) Friday

13. On what dates of August 1988 did 'Friday' falls?
1) 6, 13, 20, 27 2) 4, 11, 18, 25
3) 5, 12, 19, 26 4) 3, 10, 17, 25

14. The year next to 1973 having the same calendar as that of 1973 is _____
1) 1976 2) 1977
3) 1978 4) 1979

15. The year next to 1988 having the same calendar as that of 1988 is _____
1) 2016 2) 2010
3) 2004 4) 1999

16. Any day in April is always on the same day of the week as the corresponding day is
1) May 2) March
3) June 4) July

17. One of the following day which cannot be the last day of the century
1) Sunday 2) Monday
3) Wednesday 4) Saturday

18. What was the day on 1st January, 1 A.D.?
1) Sunday 2) Monday
3) Tuesday 4) Wednesday

19. What was the day on 31st December, 1 A.D.?
1) Sunday 2) Monday
3) Tuesday 4) Wednesday

20. How many days are there from 2nd Jan 1993 to 15th March 1993?
1) 73 2) 71
3) 37 4) 80

RACES AND GAMES OF SKILL

Race: A contest of speed over a specified distance is called race.
Eg: Running, Driving, Riding, Sailing, Rowing, etc… are the areas where the Race is concerned.

Race Course: The ground or path on which the races are arranged is called a race course.

Note:

1. The point from where a race begins is called the starting point.
2. The point where the race finishes is called the winning post or finishing point or goal.
3. The person who first reaches the goal is the winner.

Dead-heat Race: If all the contestants of a race reach the goal exactly at the same time, then the race is called a dead-heat race.

Important Note:

If P and Q are two contestants in a race, then the following statements and their corresponding meanings are very useful in solving the problems.

Statements	Mathematical Interpretation
P beats Q by t seconds	P finishes the race t seconds before Q finishes.
P gives Q a start of t seconds	P starts t seconds after Q starts from the same point.
P gives Q a start of x metres	While P starts at the starting point, Q starts x metres ahead from the starting point at the same time.
Game of 100	A game in which the participant scoring 100 points first is the winner.
In a game of 100, "P can give Q 20 points"	While P scores 100 points, Q scores only 100 – 20 = 80 points.

Formulae:

Two participants

1. If ***P*** is ***n*** times as fast as ***Q*** and ***P*** gives ***Q*** a start of ***x*** metres, then the length of the race course, so that both ***P*** and ***Q*** reach the winning post at the same time must be,

$$x\left(\frac{n}{n-1}\right) \text{ metres}$$

2. ***P*** can run ***x*** m race in t_1 seconds and Q in t_2 seconds, then ***P*** beats ***Q*** by a distance

$$\frac{x}{t_2}\left(t_2 - t_1\right) \text{ metres} \qquad [\text{Here } t_2 > t_1]$$

Three participants:

Suppose A, B, and C participate in a race. The length of the race is L meters. If A is the winner , i.e A gets the first position, in the race.

If A beats B by x meters and A beats C by y meters

◄═══════L═══════►
O-------------o---------------A
B<------------->
X mts

◄═══════L═══════►
O------------o-------------------A
C<---------------->
y mts

The value of *x* and *y* decide will decide the II position in the race.
If $x < y$ then B will beat C, i.e B gets the II position.

If $x > y$ then C will beat B, i.e C gets the II position.
The following rule is used for three participants in a race of same length.

$$(L - x_{12})\, x_{23} = L\,(x_{13} - x_{12})$$

Solved Examples

1. ***P*** is $2\frac{2}{3}$ times as fast as ***Q***. If ***P*** gives ***Q*** a start of 30 m, how long would should the race course be so that both of them reach at the same time?

Sol: Here, $n = \frac{8}{3}$ and $x = 30$

$\therefore$ Length of race course = $x\left(\frac{n}{n-1}\right)$ metres

$$= 30\left(\frac{\frac{5}{3}}{\frac{5}{3}-1}\right) = 30 \times \frac{5}{5-3} = 75 \text{ m.}$$

2. ***P*** can run 210 m in 45 sec and ***Q*** in 30 sec. By what distance ***P*** beats ***Q***?

Sol: Here, $x = 210$, $t_1 = 45$, $t_2 = 30$

$\therefore$ ***P*** beats ***Q*** by a distance = $\frac{x}{t_2}\left(t_2 - t_1\right)$

$$= \frac{210}{45}\left(45 - 30\right) = 70 \text{ m.}$$

3. ***P***, ***Q*** and ***R*** participate in a km race. If P can give Q a start of 40 m and Q can give R a start of 25 m then find how many metres P can give R a start?

Sol: Here, ***P*** is the winner (I).
Since Q can give R a start, therefore Q becomes II and R becomes III in the race.
Hence, by using the formula

$(L - x_{12})\, x_{23} = L\,(x_{13} - x_{12})$ we get,

$(1000 - 40) \times 25 = 1000 \times (- 40)$

$\Rightarrow x_{13} = 64$ metres

So, P can give R a start of 64 metres.

Exercise – 14

1. In a km race, P beats Q by 25 m or 5 sec then find the time taken by P to complete the race.
1) 3 min 15 sec 2) 4 min 20 sec

3) 2 min 30 sec　　4) 5 min 10 sec

2. In a race of 300 m P beats Q by 15 m or 5 sec. Find the P's time over the course
1) 100 sec　　2) 95 sec
3) 105 sec　　4) 90 sec

3. P can run 500 in 30 sec and Q in 35 sec. How much m start can P give to Q in a km race so that the race may end in a dead-heat?
1) $139\frac{5}{7}$　　2) $138\frac{5}{7}$
3) $142\frac{6}{7}$　　4) $140\frac{5}{7}$

4. P runs $1\frac{3}{8}$ times as fast as Q. If P gives Q a start of 120 m and they reach the goal at the same time, the goal is at a distance of
1) 360 m　　b) 440 m
3) 460 m　　d) 380 m

5. In a game of 100 points P can give Q 20 points and R 28 points. Find the number of points Q can give R in a game of 100 points.
1) 10　　2) 90
3) 15　　4) 85

6. A's speed is $1\frac{1}{2}$ times that of B. In a race A gives B a start of 300 m. How long should the race course be so that both reach the winning post simultaneously?
1) 700 m　　2) 900 m
3) 800 m　　4) 850 m

7. In a race of 600 m, P can beat Q by 60 m and in 500 m, Q can beat R by 50 m. By how many m will P beat R in 400 m race?
1) 364 m　　2) 254 m
3) 324 m　　4) 354 m

8. In a 100 m race, P runs at 5 kmph. P gives Q a start of 8 m and still betas him by 8 sec. Find the speed of Q.
1) 6.14 kmph　　2) 4.14 kmph
3) 3.14 kmph　　4) 2.14 kmph

9. In a game, P can give Q 20 points, P can give R 32 points and Q can give R 1 5points. How many points make the game?
1) 100　　2) 200
3) 300　　4) 400

10. In a game of billiards, P can give Q 6 points in 50 and can give R 13 in 65. In a game of 55, how many number of points Q can give to R?
1) 3　　2) 4
3) 5　　4) 8

11. In a km race P can beat Q by 80 m and Q can beat R by 60 m. In the same time P can beat R by
1) 135.2 m　　2) 130.5 m
3) 142 m　　4) 132.5 m

12. In a game of 90 points, P can give Q 15 points and R 30 points. How many points can Q give R in a game of 100 points?
1) 140　　2) 20
3) 50　　4) 30

13. In a race of 600 m, P can beat Q by 60 m and in a race of 500 m, Q can beat R by 50 m. By how many m will P beat R in a race of 400 m?
1) 78 m　　2) 56 m
3) 76 m　　4) 86 m

14. In a game P can give Q 25 points in 75 and R 18 points in 90. How many points can R give Q in a game of 120?
1) 20　　2) 30
3) 40　　4) 50

15. P and Q run a 5 km race on a round course of 400 m. If their speeds are in the ratio 5 : 4, how often does the winner pass the other?
1) $4\frac{1}{2}$ times　　2) $3\frac{1}{2}$ times
$2\frac{3}{4}$　　1

3) times   4) $2\frac{\ }{2}$ times

16. In a 500 m race, the ratio of speeds of two contestants P and Q is 3 : 4. P has a start of 140 m. Then, P wins by
    1) 60 m   2) 40 m
    3) 20 m   4) 10 m

17. In a km race P beats Q by 100 m and R by 200 m, by how much can Q beat R in a race of 1350 m?
    1) 150 m   2) 120 m
    3) 1200 m   4) 210 m

Experiment

ROBABILITY

An operation which results in some well-defined outcomes is called an experiment.

Random Experiment

An experiment whose outcome cannot be predicted with certainty is called a random experiment. In other words, if an experiment is performed many times under similar conditions and the outcome of each time is not the same, then this experiment is called a random experiment.

Example: a) Tossing of a fair coin
b) Throwing of an unbiased die
c) Drawing of a card from a well shuffled pack of 52 playing cards

Sample Space

The set of all possible outcomes of a random experiments is called the sample space for that experiment. It is usually denoted by S.

Example:

a) When a die is thrown, any one of the numbers 1, 2, 3, 4, 5, 6 can come up. Therefore. Sample space S = {1, 2, 3, 4, 5, 6}

b) When a coin is tossed either a head or tail will come up, then the sample space w.r.t. the tossing of the coin is
S = {H, T}

c) When two coins are tossed, then the sample space is

Sample point / event point

Each element of the sample spaces is called a sample point or an event point.

Example: When a die is thrown, the sample space is S = {1, 2, 3, 4, 5, 6} where 1, 2, 3, 4, 5 and 6 are the sample points.

Discrete Sample Space

A sample space S is called a discrete sample if S is a finite set.

Event

A subset of the sample space is called an event.

Problem of Events

Sample space S plays the same role as universal set for all problems related to the particular experiment.
ϕ is also the subset of S and is an impossible Event.
S is also a subset of S which is called a sure event or a certain event.

Types of Events

A. Simple Event/Elementary Event

An event is called a simple Event if it is a singleton subset of the sample space S.

Example:

a) When a coin is tossed, then the sample space is
S = {H, T}
Then A = {H} occurrence of head and B = {T} occurrence of tail are called Simple events.

b) When two coins are tossed, then the sample space is
S = {(H,H); (H,T); (T,H); (T,T)}
Then A = {(H,T)} is the occurrence of head on 1^{st} and tail on 2^{nd} is called a Simple event.

B. Mixed Event or Compound Event or Composite Event

A subset of the sample space S which contains more than one element is called a mixed event or when two or more events occur together, their joint occurrence is called a Compound Event.

Example:

When a dice is thrown, then the sample space is
S = {1, 2, 3, 4, 5, 6}
Then let A = {2, 4 6} is the event of occurrence of even and B = {1, 2, 4} is the event of occurrence of exponent of 2 are Mixed events

Compound events are of two type:

a) Independent Events, and
b) Dependent Events

C. Equally likely events

Outcomes are said to be equally likely when we have no reason to believe that one is more likely to occur than the other

Example: When an unbiased die is thrown all the six faces 1, 2, 3, 4, 5, 6 are equally likely to come up.

D. Exhaustive Events

A set of events is said to be exhaustive if one of them must necessarily happen every time the experiments is performed.

Example: When a die is thrown events 1, 2, 3, 4, 5, 6 form an exhaustive set of events.\

Important

We can say that the total number of elementary events of a random experiment is called the exhaustive number of cases.

E. Mutually Exclusive Events

Two or more events are said to be mutually exclusive if one of them occurs, others cannot occur. Thus if two or more events are said to be mutually exclusive, if not two of them can occur together.
Hence, $A_1, A_2, A_3, \dots$, An are mutually exclusive if and only if $A_i \leftrightarrow A_j = \phi$ ” $i \pi j$

Example:

a) When a coin is tossed the event of occurrence of a head and the event of occurrence of a tail are mutually exclusive events because we cannot have both head and tail at the same time.

b) When a die is thrown, the sample space is S = {1, 2, 3, 4, 5, 6} Let
A is an event of occurrence of number greater than 4 i.e., {5, 6}

B is an event of occurrence of an odd number {1, 3, 5}
C is an event of occurrence of an even number {2, 4, 6}
Here, events B and C are Mutually Exclusive but the event A and B or A and C are not Mutually Exclusive.

F. Independent Events or Mutually Independent events

Two or more event are said to be independent if occurrence or non-occurrence of any of them does not affect the probability of occurrence of or non-occurrence of their events.
Thus, two or more events are said to be independent if occurrence or non-occurrence of any of them does not influence the occurrence or non-occurrence of the other events.
Example: Let bag contains 3 Red and 2 Black balls. Two balls are drawn one by one with replacement.
Let A is the event of occurrence of a red ball in first draw.
B is the event of occurrence of a black ball in second draw.
then probability of occurrence of B has not been affected if A occurs before B. As the ball has been replaced in the bag and once again we have to select one ball out of 5(3R + 2B) given balls for event B.

G. Dependent Events

Two or more events are said to be dependent, if occurrence or non-occurrence of any one of them affects the probability of occurrence or non-occurrence of others.
Example: Let a bag contains 3 Red and 2 Black balls. Two balls are drawn one by one without replacement.
Let A is the event of occurrence of a red ball in first draw
B is the event of occurrence of a black ball in second draw.
In this case, the probability of occurrence of event B will be affected. Because after the occurrence of event A i.e. drawing red ball out of 5(3R + 2B), the ball is not replaced in bag. Now, for the event B, we will have to draw 1 black ball from the remaining 4(2R + 2B) balls which gets affected due to the occurrence of event A.

H. Complementary Events

Let S be the sample space for a random experiment and let E be the event. Also, Complement of event E is denoted by E′ or $\bar{E}$, where E′ means non occurrence of event E.
Thus E′ occurs if and only if E does not occur.

$\therefore n(E) + n(E') = n(S)$

Occurrence of an Event

For a random experiment, let E be an event
Let E = {a, b, c}. If the outcome of the experiment is either a or b or c then we say the event has occurred.
Sample Space : The outcomes of anytype **Event** : The outcomes of particular type

Probability of Occurrence of an event

Let S be the same space, then the probability of occurrence of an event E is denoted by p(E) and is defined as
P(E) = n(E)/n(S) = number of elements in E/number of elements in S

$$P(E) = \frac{\text{number of favourable/particular cases}}{\text{total number of cases}}$$

Example:

a) When a coin is tossed, then the sample space is S = {H, T}
Let E is the event of occurrence of a head
⇒ E = {H}

b) When a die is tossed, sample space S = {1, 2, 3, 4, 5, 6}
Let A is an event of occurrence of an odd number
And B is an event of occurrence of a number greater than 4
⇒ A = {1, 3, 5} and B = {5, 6}
∴ P(A) = Probability of occurrence of an odd number = n(A)/n(S) = 3/6 = ½
and P(B) = Probability of occurrence of a number greater than 4 = n(B)/n(S) = 2/6 = 1/3

EXERCISE-15

Questions 1-7

A coin is flipped three. Find the probability of getting
1.A head exactly once.
1.1/8 2.1/ 4 3.3/8 4.1/ 2
2.tails exactly twice.
1.1 /4 2.3/8 3.1/8 4.None of these
3.heads all three times.
1.1/8 2.¼ 3.3/8 4.1/ 2
4.a tail at least once.
1.1 /4 2.7/8 3.1/8 4.3/8
5.a head at least two times. S
1.3/8 2.1 /4 3.1/8 4.1 /2
6.a head in the first throw, a tail in the second, and a head in the third.
1.1 /8 2.1/ 4 3.3/8 4.1 /2
7.a head in the third toss, if in the first two tosses the coin landed tails.
1.1/ 2 2.1/ 8 3.7/8 4.1 /4

Questions 8 – 11

From a pack of 52 cards, a card is chosen at random. Find the probability that
9. a card of hearts.
1.1 /2 2.1 /4 3.1 /2 4.1/52
10.a 7 of clubs.
1.1/26 2.5/52 3.1/13 4.1/52
11.a king of diamonds or hearts.
1.1/13 2.3/26 3.1/26 4.1 /4

Questions 12 – 16

A bag contains 5 red, 6 blue and 9 black balls. Find the probability that a ball drawn at random

12.is either red or blue or black.

1.0 2.19/20 3.21/20 4.20/20

13.is blue.

1.1 /4 2.7/10 3.3/10 4.9/20

14.is red or blue.

1.11/20 2.1 /4 3.3 /4 4.7/10

15.is blue or black.

1.11/20 2.1 /4 3.3 /4 4.7/10

16.is not blue.

1.1 /4 2.7/10 3.3/10 4.9/20

17. Two cards are drawn successively with replacement from a well shuffled deck of 52 cards, the probability that both of these will be aces is:

1.1/169 2.1/201 3.1/2652 4.4/663

18. Find the probability that there are 53 Mondays in Leap year.

1.1/7 2.2/7 3.3/7 4.4/7

19.if P (A)=0.3 P(B)=0.4 and □B)=0.6, then find P(A□B).

1.0.3 2.0.4 3.0.1 4.0.5

20. In throwing a fair dice, what is the probability of getting the number '3'?

1.1/3 2.1/6 3.1/9 4.1/12

21. What is the number of throwing a number greater than 4 with an ordinary dice whose faces are numbers from 1 to 6.

1.1 /3 2.1/6 3.1/9 4.1/12

22. Three coins are tossed. What is the probability of getting

(i)2 Tails and 1 Head

1.1 /4 2.3/8 3.2/3 4.1/ 8

(ii)1 Tail and 2 Heads

1.3 /8 2.1 3.2 /3 4.3 /4

23. Three coins are tossed. What is the probability of getting

(i)neither 3 Heads nor 3 Tails

1.1 /2 2.1 /3 3.2/3 4.3 /4

(ii)three heads

1.1 /8 2.1 /4 3.1 /2 4.2/ 3

24. What is the probability of throwing a number greater than 2 with a fair dice

1.2 /3 2.2/5 3.1 4.3/5

25.A can hit the target 3 times in 6 shots, B 2 times in 6 shots and C 4 times in 6 shots. They fire a volley. What is the probability that at least 2 shots hit?

1.1 /2 2.1/3 3.2 /3 4.3/ 4

Types of Plane Figures

MENSURATION

1. Triangle 2. Quadrilateral 3. Polygon 4. Circle 5. Sector of a circle 6. Rectangular Paths 7. Circular paths

I. Triangle

(a). Any triangle

a, b and c are three sides of the triangle; h is the altitude and AC is the base.

Perimeter (P) = a + b + c = 2s

Area (A) = (1/2)xbasexaltitude =(1/2)xany sidexlength of perpenticular dropped on that side = $\sqrt{s(s-a)(s-b)(s-c)}$

(b). Equilateral

a is the length of each side

Perimeter (P) = 3a , **Area (A)** = $\frac{\sqrt{3}}{4}$ a2

(c). Right-angled

b, c are the lengths of the two legs

Perimeter (P) = a + b + c = 2s

Area (A) = $\frac{1}{2}\times$ basexheight = $\frac{1}{2}\times$ cxb

(d). Isosceles

a is the length of two equal sides

b is the base

BD is the perpendicular dropped on base such that it divides the base equally AD = CD = $\frac{b}{2}$, **Perimeter (P)** = 2a+b

Area (A) = $\frac{b}{4}\sqrt{4a^2-b^2}$

(e). Right-angled Isosceles

Perimeter (P) = $\sqrt{2}$ a($\sqrt{2}$ +1) ,**Area (A)=** $\frac{1}{2}\times$ $(a)^2$

II. Quadrilateral

(a). Any Quadrilateral

AC is the diagonal = d, DE and BF are two perpendiculars drawn on the diagonal (AC) P_1, and P_2 are the lengths of the two perpendiculars

Perimeter (P) = sum of the four sides.

Area (A) = $\frac{1}{2}\times$ d (p_1+p_2) = $\frac{1}{2}\times$ any diagonal $\times$ (sum of $\perp$ rs drawn on that diagonal)

(b). Rectangle

l = length

b = breadth d

= diagonal

Perimeter (P) : P = 2(l + b) = 2(l+$\sqrt{d^2-l^2}$) ,

Area (A): A = l$\times$ b = l $\times$ $\sqrt{d^2-l^2}$

(c). Square

a = length of side

d = diagonal

Perimeter (P) : P = 4a = 2d $\sqrt{2}$

Area (A) : A = a^2 = $\frac{d^2}{2}$ = $\frac{p^2}{16}$

d). Rhombus

a = each side

d_1 = one diagonal

d_2 = another diagonal h

= height

Perimeter (P) : P = 4a = 2$\sqrt{d_1^2+d_2^2}$

Area (A) : A = $\frac{1}{2}\times$ $d_1\times d_2$

= = = a× h

(e). Trapezium

a and b are two parallel sides, h is the height

Area (A) : $A = \frac{1}{2}(a + b) \times h = \frac{1}{2} \times$ (sum of parallel sides) $\times$ (perpendicular distance between parallel sides)

(f). Parallelogram

b is the base

h is the perpendicular distance between the base and its opposite side

Area (A) : $A = b \times h$ = base $\times$ (perpendicular distance between the base and its opposite sides)

= 2 $\times$ area of Δ ABD (or Δ BCD)

<u>III.</u> <u>Polygon</u>

Polygon is a n-sided closed figure bounded only by line segments.

In a polygon if the internal angle at each vertex is less than 180° then the polygon is a convex polygon, else a concave polygon.

Convex Polygon:

Area of a regular polygon $= \frac{1}{2} \times$ perimeter $\times \perp^r$ distance from the center of the polygon to any side.

Number of diagonals in a polygon $= \frac{n(n-3)}{2}$

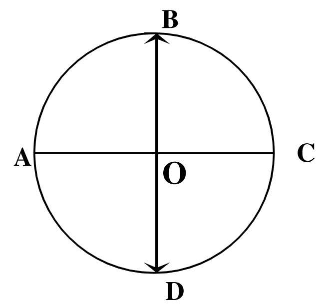
Sum of all interior angles of a polygon = $(2n-4) \times 90^\circ$

Each interior angle of n-sided regular polygon $= \left[\frac{n-2}{n}\right] \times 180^\circ$

Sum of all exterior angles of n-sided regular polygon = 360°

Each exterior angle of n-sided regular polygon $= \frac{360}{n}$

<u>IV.</u> <u>Circle</u>



O is the center of the circle

OA = OC = OB = OD = radius of circle = r

AC = BD = diameter of circle = d = 2r

Circumference (or Perimeter) C = $2\pi r = \pi d$

Area of circle (A) $= \pi r^2 = \pi \frac{d^2}{4}$

If C = circumference, A = area then

A $= \frac{C^2}{4\pi}$ and $\frac{A}{C} = \frac{r}{2}$

Examples:

1. If three sides of a triangle are 5, 6 and 7 cm respectively, find the area of triangle.

Sol: Area of $\Delta = \sqrt{s(s-a)(s-b)(s-c)}$

Now, $s = \frac{a+b+c}{2} = \frac{5+6+7}{2} = 9$

$\therefore$ Area $= \sqrt{9 \times (9-5)(9-6)(9-7)} = \sqrt{9 \times 4 \times 3 \times 2}$

$= \sqrt{216} = 6\sqrt{6}$ cm^2.

2. ABC is an equilateral triangle of side 24 cm. Find the in radius of the triangle.

Sol: In a equilateral triangle, the altitude, median and perpendicular are equal.

$\therefore$ AD = $\sqrt{3}/2 \times 24 = 12\sqrt{3}$

GD (in radius) = $1/3 \times 12\sqrt{3} = 4\sqrt{3}$ cm

3. The base and other side of an isosceles triangle is 10 and 13 cm respectively. Find its area.

Sol: Area of Isosceles $\Delta = \frac{b}{4}\sqrt{4a^2 - b^2}$

Given, base b = 10 Other side a = 13

Area (A) $= \frac{10}{4}\sqrt{4\times(13)^2-10^2} = \frac{10}{4}\sqrt{676-100}$

$= \frac{10}{4} \times 24 = 60 \text{ cm}^2$.

4. In a right-angled triangle, the length of two legs are 12 and 5 cm. Find the length of hypotenuse and its area.

Sol: In a right angled triangle, $(\text{Hypotenuse})^2 =$ $(\text{one leg})^2 + (\text{other leg})^2$

$= 12^2 + 5^2$

$\therefore$ Hypotenuse $= \sqrt{12^2+5^2} = \sqrt{169} = 13$ cm.

In a right angled triangle,

Area $= \frac{1}{2} \times (\text{leg})_1 \times (\text{leg})_2 = \frac{1}{2} \times 12 \times 5 = 30\text{cm}^2$.

If the perimeter and diagonal of a rectangle and 14 and 15 cm respectively. Find its area.

Sol: In a rectangle,

$\frac{(\text{Perimeter})^2}{4}$ = (diagonal)2 + 2 x Area ; $\frac{(14)^2}{4}$ = $(5)^2$ + 2 x Area

$\Rightarrow$ 2 x Area $= \frac{196}{4}$ - 25 $\therefore$ Area $= \frac{49-25}{2}$ = 12 cm^2.

5. Find the length of the diagonal and the perimeter of a square plot if its area is 900 square metres.

Sol: In a square, A $= \frac{d^2}{2} = \frac{p^2}{16}$ (Diagonal)^2

$\therefore$ = 2 x Area = 900

$\therefore$ Diagonal (d) = $\sqrt{2\times900} = 30\times\sqrt{2}$ = 42.42 metres

$(\text{Perimeter})^2$ = 16 x Area = 16 x 900

$\therefore$ Perimeter (P) $= \sqrt{16\times900}$ = 120 metres.

7. A field in the shape of a rhombus has the distances between pairs of opposite vertices as 14 m and 48 m. What is the cost (in rupees) of fencing the field at Rs.20 per metre?

Sol: The diagonals are 14 m and 48 m

Sides of rhombus $= \sqrt{\left(\frac{14}{2}\right)^2 - \left(\frac{48}{2}\right)^2} = \sqrt{625}$ = 25

Perimeter of rhombus = 4 x 25 = 100 m.

Cost of fencing the field = 100 x 20 = Rs.2000

8. In a trapezium, the length of parallel sides are 20 and 25 metres respectively and the perpendicular distance between the parallel sides is 12 metres. Find the area of trapezium.

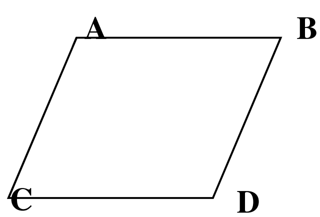
Sol: One parallel side a = 20 metres. Second parallel side b = 25 metres. Height (perpendicular distance between a and b) = 12 metres.

Area $= \frac{1}{2}(a+b)\times h = \frac{1}{2}(20+25)\times 12 = 270\text{m}^2$.

9. The distance between a pair of opposite vertices of a quadrilateral is 32 units. The lengths of the perpendiculars drawn on to this diagonal from the other two vertices are 4 1/3 units and 6 2/3 units respectively. Find the area (in sq units) of the quadrilateral?

Sol: Area of quadrilateral = 1/2 x 32 x$\left(\frac{13}{3}+\frac{20}{3}\right)$ = 178 sq units.

9.



In the above parallelogram ABCD, $\angle A = x + 30°$ and $\angle D = x - 40°$, what is the measure of $\angle DCB$?

Sol: In a parallelogram, sum of adjacent angles is equal to 180° x

+ 30 + x − 40 = 180 $\qquad$ x = 95°

$\angle DAB$ = x + 30 = 95 + 30 = 125°

$\therefore$ $\angle DCB$ = $\angle DAB$ = 125°

(opposite angles of a parallelogram are equal)

10. In a circle of radius 49 cm, an arc subtends an angle of 36° at the centre. Find the length of the arc and the area of the sector.

Sol: Length of the arc $= \frac{2\Pi r\theta}{360} = \frac{2 \times 22 \times 49 \times 36}{7 \times 360}$ = 30.8 cm

Area of the sector $= \frac{\Pi r^2\theta}{360} = \frac{22 \times 49 \times 49 \times 36}{7 \times 360}$ = 754.6 cm^2

11. A rectangular plot of dimensions 13 m x 17 m is surrounded by a garden of width 5 m. What is the area (in sq m) the garden?

Sol: Let ABCD be the rectangular plot of given dimension. The shaded part is the surrounding garden. Now, the plot ABCD together with the garden forms another rectangular form PQRS. Dimensions of PQRS, as can be seen from the diagram, are:

Length PQ = width of garden + AB + width of garden

= 5 + 17 + 5 = 27 m

Similarly, breadth = PS = 5 + 13 + 5 = 23 m

Area of garden = Area of PQRS − Area of ABCD

= (27 x 23) − (17 x 13) = 621 − 221 = 440 sq m.

12. There is a rectangular field of length 100 m and breadth 40 m. A carpet of 2 m width is to be spread from the centre of each side to the opposite side. What is the area of the carpet?

Sol: Area of the carpet ABCD = 40 m x 2 m = 80 m^2

Area of the carpet EFGH = 100 m x 2 m = 200 m^2

But the common area of two carpets = 2 x 2 = $4m^2$

So, area of the carpet = 200 + 80 − 4 = 276 m^2

<u>Exercise:-15</u>

1. The base and other side of an isosceles triangle is 10 cm and 13 cm respectively. Find its area. 1. 23 cm^2 2. 60 cm^2 3. 65 cm^2 4. 23 cm^2
2. If the area of triangle is 150 m^2 and base : height is 3 : 4, find its height and base respectively. 1. 75 m, 100 m 2. 100 m, 75 m 3. 75 m, 75 m 4. None
3. Find the area of an equilateral triangle of side of 12 cm.
   1. 72 sq cm 2. 36 $\sqrt{3}$ sq cm 3. 12 $\sqrt{3}$ sq cm 4. 18 $\sqrt{3}$ sq cm
4. The height of a triangle is 8/9th of its base and its area is 576 sq cm. Find its height. 1. 36 cm 2. 52 cm 3. 72 cm 4. 32 cm
5. Find the area of a triangle whose sides are 66 cm, 88 cm and 1.1 m.
   1. 2640 sq cm 2. 2904 sq cm 3. 2940 sq cm 4. 1452 sq cm
6. Area of an equilateral triangle is 16 $\sqrt{3}$ sq cm, Find its perimeter.
   1. 12 cm 2. 48 cm 3. 24 cm 4. 16 cm
7. What is the height of an equilateral triangle if its side is $8\sqrt{3}$ cm?
   1. 6 cm 2. 8 cm 3. 24 cm 4. 12 cm
8. In a quadrilateral, the length of its diagonals is 12 cm and the offsets drawn on this diagonal measure 13 cm and 7 cm respectively. Find its area.
   1. 546 m^2 2. 273 m^2 3. 60 m^2 4. 120 m^2
9. The base and the height of a parallelogram are 25 cm and 20 cm respectively. Find its area. 1. 500 sq cm 2. 250 sq cm 3. 45 sq cm 4. 125 sq cm
10. If the perimeter and diagonal of a rectangle and 14 cm and 5 cm respectively. Find its area.
    1. 6 cm^2 2. 19 cm^2 3. 12 cm^2 4. 9 cm^2
11. The area and the perimeter of a rectangle are 84 m^2 and 38 m respectively. Find its length and breadth.
    1. 12 m, 7 m 2. 14 m, 6 m 3. 42 m, 19 m 4. None
12. A rectangular grass field is 112 m x 78 m. It has a gravel path 2.5 m wide all round it on the inside. Find the area of gravel path.
    1. 8736 sq m 2. 925 sq m 3. 4368 sq m 4. 952 sq m
13. The length of a rectangle is increased by 20% and the breadth is decreased by 30%. Find the percentage change in its area.
    1. 10% increase 2. 16% decrease 3. 8% decrease 4. 16% increase
14. The length and the breadth of a rectangle are in the ratio of 15 : 8 and its perimeter is 230 cm. Find its area.
    1. 3000 sq cm 2. 2300 sq cm 3. 1500 sq cm 4. 6000 sq cm

15. There is a path of 1 m width around the outside of a rectangular field of 98 m x 48 m. Find the area of the path.
1. 148 sq m 2. 296 sq m 3. 598 sq m 4. 2352 sq m

16. The breadth of a rectangle is 4/5th of its length and its area is 720 sq cm. Find its length. 1. 15 cm 2. 30 cm 3. 60 cm 4. 576 cm

17. The sides of a rectangle are in the ratio 4 : 3 and its area is 768 sq m. Find its perimeter? 1. 56 m 2. 112 m 3. 96 m 4. None

18. The perimeter of a rectangle is 216m. If its sides are in the ratio 5 : 4 the area is __________
1. 1140 sq m 2. 2880 sq m 3. 960 sq m 4. 1260 sq m

19. Find the length of the diagonal of a square plot if its area is 900 sq m.
1. $10\sqrt{2}$ m 2. $15\sqrt{2}$ m 3. $30\sqrt{2}$ m 4. $9\sqrt{2}$ m

20. Find the perimeter of a square plot if its area is 1600 sq m.
1. 80 m 2. 160 m 3. 320 m 4. 40 m

21. Find the ratio of area and the perimeter of a square of side 8 cm.
1. 1 : 2 2. 4 : 1 3. 3 : 1 4. 2 : 1

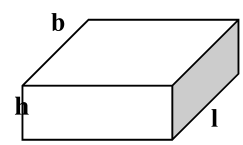
22. Find the diagonal of a square whose perimeter is 128 $\sqrt{2}$ sq m.
1. 64 m 2. 32 m 3. $32\sqrt{2}$ m 4. $64\sqrt{2}$ m

23. The perimeter of a square is 88 cm. Find its area.
1. 484 sq cm 2. 174 sq cm 3. 242 sq cm 4. None

24. There is a square shaped grass lane of 14 m side. Four cows are tethered with the ropes of 3.5 m length each at one corner. Find the area of the grass lane over which the cows are unable to graze the grass.
1. 157.5 sq m 2. 38.5 sq m 3. 175.5 sq m 4. 157.7 sq m

25. The area of two squares is in the ratio of 16 : 49. Find the ratio of their diagonals.
1. 7 : 4 2. 49 : 16 3. 4 : 7 4. None

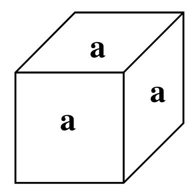
<u>Cuboid :</u>



A right prism with a rectangular base is called a ***Cuboid.***
The sides of the base are length (l) and breadth (b). The height is h.

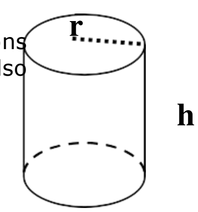
Lateral Surface Area = 2h(l + b)
Total Surface Area = 2h(l + b) + 2lb = 2(lb + bh + hl)
Longest diagonal $=\sqrt{l^2+b^2+h^2}$
Volume = lbh

<u>Cube:</u>



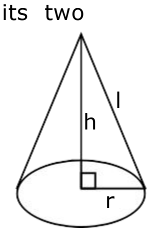
If the length, breadth and height of a cuboid are all equal, it is called a ***cube.***
Then, if edge of the cube = a
Longest diagonal = $\sqrt{3}$ a
Lateral Surface Area $=6a^2$
Total surface Area $=6a^2$
Volume $= a^3$

<u>Cylinder :</u>



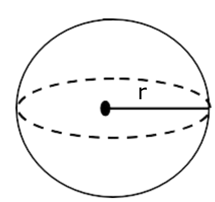
A cylinder can be considered to be a right prism except that instead of identical polygons a cylinder has identical circles for its top and base and it has a single lateral surface also called curved surface, instead of several rectangular surfaces.
The basic measurements are the radius of the base (or top) r and the height h.
Curved Surface (Lateral Surface Area) = 2π rh
Total surface Area = 2πrh+2 πr² = 2πr(h + r)
Volume = π r^2h

<u>Cone:</u> A cone can be formed from the sector of a circle by rolling it and joining together its two straight edges. If r is the radius of the cone, and R is the radius of the sector of angle θ , then



1. $r = \frac{\theta}{360} \times R$
2. Relation between r, l and h. (the radius, the slant height and height) is $l^2 = h^2+r^2$
3. Curved Surface area of Cone = πrl
4. Total Surface Area = π rl + π r^2 = π r(l + r)
5. Volume $= \frac{1}{3}$ π r^2h

<u>Sphere:</u>



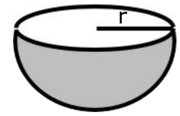
All points on the surface of a sphere are at the same distance from the center of the sphere.
This distance is called the radius, r.
Surface Area of Sphere = 4 πr^2
Volume of a Sphere $= \frac{4}{3}$ π r^3
The sphere has only one surface and hence only one surface area.

<u>Hemisphere:</u>

The radius is r.

Curved Surface Area = $2\pi r^2$

Total Surface Area = $2\pi r^2 + \pi r^2 = 3\pi r^2$



Volume $= \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$

Examples

1. A cuboid is 20 m x 10 m x 8 m. Find the length of diagonal, surface area and volume.

Sol: In a cuboid ,

Diagonal d $= \sqrt{l^2 + b^2 + h^2} = \sqrt{20^2 + 10^2 + 8^2}$ = 23.75

Surface are S = 2 (20 x 10 + 10 x 8 + 8 x 20) = 880 m^2

Volume = l x b x h = 20 x 10 x 8 = 1600 m^3.

2. A cube has edge 12 m. Find its length of diagonal, surface area and volume.

Sol: In a cube

Diagonal d = Edge x $\sqrt{3}$ = 12 x $\sqrt{3}$ = 20.78 m

Surface area S = 6 x $(Edge)^2$ = 6 x $(12)^2$ = 864 m^2

Volume V = $Edge^3 = (12)^3$ = 1728 m^3

3. The base of a right prism is a regular pentagon of side 18 cm. If the height of the prism is 2/3rd of the side of the base, how much is the lateral surface area (in sq cm) of the prism?

Sol: Perimeter of the base of the prism

= number of sides x length of each side

= 5 x 18 = 90 cm.

Lateral surface area of a right prism = (Base perimeter) x (height)

$= (90)\left(\frac{2}{3} \times 18\right)$ = 1080 sq cm

4. If the radius of a sphere is increased by 50%, find the increase percent in volume and the increase percent in the surface area

Sol: Let original radius = R. Then new radius = $\frac{150}{100}R = \frac{3R}{2}$.

Original volume $= \frac{4}{3}\Pi R^3$, New volume $\frac{4}{3}\Pi\left(\frac{3R}{2}\right)^3 = 9\Pi R^3$.

Increase % in volume = $\left(\frac{19}{6}\Pi R^3 \times \frac{3}{4\Pi R^3} \times 100\right)$% = 237.5%

Original surface area = $4\Pi R^2$. New surface area = $4\Pi\left(\frac{3R}{2}\right)^2 = 9\Pi R^2$.

Increase % in surface area = $\left(\frac{5\Pi R^2}{4\Pi R^2} \times 100\right)$ % = 125%.

5. A cylinder with base radius of 8 cm and height of 2 cm is melted to form a cone of height 6 cm. Find the radius of the cone?

Sol: Let the radius of the cone be r cm

Then, $\frac{1}{3} \times \Pi \times r^2 \times 6 = \Pi \times 8 \times 8 \times 2 \Leftrightarrow r^2 = \left(\frac{8 \times 8 \times 2 \times 3}{6}\right) = 64 \Leftrightarrow$ r = 8 cm.

What is the maximum length of a pencil which can be inscribed in a box of length 24 units, breadth 3 units and height 4 units?

Sol: Maximum length in a cuboid is its diagonal

∴ Length of main diagonal is $\sqrt{length^2 + breadth^2 + height^2}$

$= \sqrt{(24)^2 + (3)^2 + (2)^2} = \sqrt{576 + 9 + 4} = \sqrt{589}$ units

6. The height and base-radius of a right circular cone are 10 cm and 24 cm respectively. What is the area of the curved surface area (in sq cm) if the cone?

Sol: Curved surface area of a cone $= \Pi$ rl,

R and l being radius and slant height.

It is given that height h = 10 cm and radius = 24 cm.

L2 = h2 + r2 = 102 +242

∴ l = 26 (10 and 24 are in the ratio of 5 : 12; hence l will be the 2 x 13 = 26

Hence, curved surface area $= \Pi$ rl $= \Pi$ x 24 x 26 624Π sq cm.

Exercise:-16

1. The surface area of cube is 96 sq cm. Find its volume.

1. 48 cm^3 2. 64 cm^3 3. 16 cm^3 4. 32 cm^3

2. The volume of a cube is 125 cm^3. Find its surface area.

1. 25 cm^2 2. 375 cm^2 3. 150 cm^2 4. 250 cm^2

3. The diagonal of a cube is 3 cm. Find its surface area.

1. 12 cm^2 2. 102 cm^2 3. 18 cm^2 4. 36 cm^2

4. A cube of 6 cm side melted and smaller cubes of 2 cm side are manufactured. Find the number of smaller cubes so formed.

1. 12 2. 27 3. 24 4. 8

5. Two cubes have their volumes in the ratio 8 : 27. The ratio of their surface areas is ___________

1. 2 : 3 2. 9 : 4 3. 2 : 9 4. 4 : 9

6. A cube of side 6 cm is cut into a number of cubes, each of side 3 cm. Find the number of cubes.

1. 8 2. 9 3. 24 4. 5

7. The percentage increase in the surface area of a cube when each side is doubled is ______________

1. 100% 2. 200% 3. 300% 4. 400%

8. The length, breadth & height of a cuboid are in the ratio of 4 : 3 : 2 and its volume is 3000 m^3. Find its surface area.

1. 1300 m^2 2. 1500 m^2 3. 1333 m^2 4. 27000 m^2

9. Two cubes each with 6 cm edge are joined end to end. The surface area of the resulting cuboid is

1. 360 cm^2 2. 36 cm^2 3. 216 cm^2 4. 360 m^2

10. Find the area of the four walls of a room of 6 m x 4 m x 3 m.

1. 120 m^3 2. 84 m^3 3. 42 m^3 4. 60 m^3