CRT STUDY MATERIAL FOR

DADI INSTITUTE OF ENGINEERING AND TECHNOLOGY, ANAKAPALLI

QUANTITATIVE APTITUDE
FOR
CAMPUS RECRUITMENT TRAINING
(CRT)

CONTENTS

NUMBER SYSTEM

Natural Numbers (N): Counting numbers 1, 2, 3, ... are called Natural numbers. They are also called Positive Integers.

N = {1, 2, 3, ...}

Whole Numbers (W): All the natural numbers including 0 together constitute the set of Whole numbers.

W = {0, 1, 2, 3, ...}

Integers (I or Z): All the whole numbers including negative counting numbers together

constitute the set of Integers.

I or Z = {. . . . . ., -3,-2,-1, 0, 1, 2, 3 . . . . . .}

Rational Numbers (Q): Numbers which are in the form of $\frac{p}{q}$, where p, q are integers and $q \neq 0$, are called Rational numbers.

$$Q = \left\{ \frac{p}{q} (q \neq 0),\ p, q \in Z \right\}$$

Eg: -3, 1, 3.2, $\frac{1}{3}, \frac{22}{7}$, etc.

Note:

1. Rational numbers are divided into two groups, namely integers and non-integers.
2. Non-integer belonging to the set of rational numbers is called fraction.

Fraction: A number expressed in the form of $\frac{p}{q}$ is also called fraction, where 'p' is the numerator and 'q' is the denominator. Fraction is a part of an integer.

Eg: $\frac{6}{5}, \frac{2}{7}, -\frac{1}{6}$, etc.

Proper Fraction: Fractions in which Numerator < Denominator are called Proper Fractions.

Eg: $\frac{1}{5}, \frac{3}{7}, \frac{7}{9}$, etc.

Improper Fraction: Fractions in which Numerator > Denominator are called Improper Fractions.

Eg: $\frac{8}{3}, \frac{7}{5}, \frac{9}{4}$, etc.

Mixed Fraction: It has two parts. One is integer and the other is a fraction.

Eg: $1\frac{1}{3}, 2\frac{3}{5}, 5\frac{4}{3}$, etc.

Note:

1. All the mixed fractions can be converted into improper fractions.
2. A rational number can be expressed in the decimal form.
3. The decimal form of a rational number is either recurring or a terminating decimal.
   Eg: 10/3 = 3.3333… (recurring)
   3/4 = 0.75 (terminating)

Irrational Numbers (Q'): A number which cannot be expressed in the form of rational number is called an Irrational number.

For an irrational number, the decimal part is non-recurring and non-terminating.

Eg: √2 = 1.414…. It is non-recurring and non-terminating.

Even number: An integer divisible by 2 is called an Even number.

Eg: 2, 4, 6, 8,……..

Odd Number: An integer not divisible by 2 is called an Odd number.

Eg: 1, 3, 5, 7,…….

Prime Numbers: Numbers which are not divisible by any number other then 1 and itself are called Prime numbers.

Eg: 2, 3, 5, 7,…….

Composite Numbers: Except 1, the numbers which are not prime are called Composite numbers

Eg: 4, 6, 9, 12,……

Co-prime Numbers: Numbers which do not have any common factor other than 1 are called Co-prime numbers.

Eg: (4, 15), (9, 22), (12, 29),……

Note:

1. 1 is neither prime nor composite.
2. 2 is an even prime number.

3. Co-prime numbers can be prime or composite numbers.
4. Any two prime numbers are always Co-prime numbers.
5. Any two consecutive positive integers are always co-primes.

Place Value of a Digit in a Numeral: The value of where the digit is in the number, such as units, tens, hundreds, etc.

Face Value: Face Value of a number is the number itself.
Consider the number 12654:

Place Value of 4 = 4 ones = 4, Face Value of 4 = 4
Place Value of 5 = 5 tens = 50, Face Value of 5 = 5
Place Value of 6 = 6 hundreds = 600, Face Value of 6 = 6
Place Value of 2 = 2 thousands = 2000, Face Value of 2 = 2
Place Value of 1 = 1 ten thousands = 10,000, Face Value of 1 = 1

Perfect Numbers: If the sum of the factors of a given number is twice the number, the number is said to be a Perfect number.
Eg: Factors of 6 = 1, 2, 3, 6 and 1 + 2 + 3 + 6 = 12
28, 496, etc….are the other examples of perfect numbers.

MULTIPLICATION TIPS:

1. For multiplication of a given number by 9, 99, 999, etc., that is by $10^n - 1$, the easy way is:
   Put as many zeros to the right of the multiplicant as there are nines in the multiplier and from the result subtract the multiplicant and get the answer.

   Eg: Multiply 2893 by 99.

   Sol: 2893 x 99 = 2893 (100 – 1) = 289300 – 2893 = 286407.

2. For multiplication of a given number by 11, 101, 1001, etc., that is by $10^n + 1$, the easy way is:
   Place *n* zeros to the right of the multiplicant and then add the multiplicant to the number so obtained.

   Eg: Multiply 3782 by 11.

   Sol: 3782 x 11 = 3782 (10 + 1) = 37820 + 3782 = 41602.

3. For multiplication of a given number by 15, 25, 35, etc.
   Double the multiplier and then multiply the multiplicant by this new number and finally divide the product by 2.

   Eg: Multiply 5054 x 15 = ½ (5054 x 30) = ½ (151620) = 75810

4. For multiplication of a given number by 5, 25, 125, 625, etc., that is, by a number which is some power of 5.
   Place as many zeros to the right of the multiplicant as is the power of 5 in the multiplier, then divides the number so obtained by 2 raised to the same power as is the power of 5.

   Eg: 2982 x 5 = 29820/2 = 14910
   5739 x 25 = 573900/2^2 = 143475

a) No. of factors of a given number: If $N = a^p \times b^q \times c^r$ ..... then the number of factors of N = $(p + 1)$ $(q + 1)$ $(r + 1)$……, where a, b, c are prime factors of N and $p, q, r,$……. are positive integers.

Eg: Find the number of factors of 24.
Sol: 24 = $2^3 \times 3^1$
∴ The number of factors of 24 = (3 + 1) (1 + 1) = 8.

b) Sum of the factors of a given number: If $N = a^p \times b^q \times c^r$ ..... then the sum of the factors of N = $\frac{a^{p+1}-1}{a-1} \times \frac{b^{q+1}-1}{b-1} \times \frac{c^{r+1}-1}{c-1}$ ..... where a, b, c are prime factors of N and $p, q, r,$……. are positive integers.

Eg: Find the sum of the factors of 24.
Sol: 24 = $2^3 \times 3^1$

$\therefore$ Sum of the factors of 24 = $\frac{2^{3+1}-1}{2-1}\times\frac{3^{1+1}-1}{3-1}=60.$

c) No. of ways of expressing a given number as a product of two factors:

If $N=a^p\times b^q\times c^r.....$ where a, b, c are prime factors of N and p, q, r,……. are positive integers then the number of ways in which N can be expressed as product of two factors =

$$\frac{1}{2}\{(p+1)(q+1)(r+1).....\}.$$

Eg: Find the number of ways of expressing 48 as a product of two factors.

Sol: 48 = $2^4\times3^1$

No.of ways = $\frac{1}{2}\{(p+1)(q+1)\}=\frac{1}{2}\{(4+1)(1+1)\}=5.$

d) No. of ways of expressing a given number which is a perfect square as a product of two factors:

If $N=a^p\times b^q\times c^r.....$ where a, b, c are prime factors of N and p, q, r,……. are positive integers then the number of ways in which N can be expressed as product of two factors =

$$\frac{1}{2}\{(p+1)(q+1)(r+1).....+1\}.$$

Eg: Find the no. of ways of expressing 36 as a product of two factors.

Sol: 36 = $2^2\times3^2$

No. of ways = $\frac{1}{2}\{(p+1)(q+1)+1\}=\frac{1}{2}\{(2+1)(2+1)+1\}=5.$

TIPS ON SQUARES:

Condition	Method	Example
To square any number ending with 5.	$(a5)^2=\{a(a+1)\}25$	$(35)^2=\{3(3+1)\}25$ = 1225
To square a number in which every digit is one.	Count the number of digits in the given number and start writing numbers in ascending order from one to this number and then in descending order up to one.	$(11)^2$ = 121, $(111)^2$=12321
To square a number which is nearer to 10 x.	Use the formula: $x^2=(x^2-y^2)+y^2=(x+y)(x-y)+y^2$	$(1004)^2$ = (1004 – 4) (1004 + 4) + $(4)^2$ = 1000(1008) + 16 = 1008016

DIVISION:

Dividend = (Divisor x Quotient) + Remainder

Eg:

$$\begin{array}{r} 3 \\ 3\overline{)10} \\ 9 \\ \hline 1 \end{array}$$

→ Quotient; → Divisor; → Dividend; → Remainder

TESTS OF DIVISIBILITY

Divisibility by…	Rule	Example	Explanation
2	Unit's digit of the number should be zero or divisible by 2.	4, 12, 102, etc.	
3	Sum of the digits in the number should be divisible by 3.	1782	1+7+8+2 = 18 which is divisible by 3 hence 1782 also divisible by 3.
4	Number formed by the last two digits should be divisible by 4 or are both zero.	4784, 300, etc.	4784 ⓒ Since 84 is divisible by 4, 4784 is also divisible by 4.

5	Unit's digit of the number should be 0 or 5.	120, 625, etc.	
6	Should satisfy divisibility rules of 2 and 3.	4518	
7	The unit digit of the given number is doubled and then it is subtracted from the number obtained after omitting the unit digit. If the result is divisible by 7, then the given number is also divisible by 7.	448	448 ⟶ 44 – 8(2) = 44 – 16 = 28 which is divisible by 7 and hence 448 is also divisible by 7.
8	Number formed by the last three digits should be divisible by 8. or zero's	1576	1576 ⟶ 576 is divisible by 8 and hence 1576 is also divisible by 8.
9	Sum of the digits in the number should be divisible by 9.	1395	1395 ⟶ 1+3+9+5 = 18 is divisible by 9 and hence 1395 is also divisible by 9.
10	Number should end in zero.	1000	
11	Sum of digits at odd places – Sum of digits at even places should be 0 or divisible by 11.	38797	38797 ⟶ Sum of digits at odd places = 3+7+7 = 17 Sum of digits at odd places = 8+9 =17 and 17 – 17 =0, hence 38797 is divisible by 11.
25	Last two digits in the number should be 0 or divisible by 25.	175	175 ⟶ 75 is divisible by 25 and hence 175 is also divisible by 25.
125	Last three digits in the number should be 0 or divisible by 125.	2250	2250 ⟶ 250 is divisible by 125 and hence 2250 is also divisible by 125.

Steps to find whether a given number is prime number or not:

1. Find the least positive integer, a such that a^2 > given number.
2. Test the divisibility of given number by every prime number that is less than a.
3. The given number is a prime number only if it is not divisible by any of these primes.

Eg: Check whether 923 is a prime number or not?

1. 923 lies between 900 and 961 which are perfect squares having square roots 30 and 31 respectively.
2. Prime numbers less than 31 are 2,3,5,7,11,13,17,19,23,29.
3. 923 is divisible 13 and hence it is not a prime number.

a) To find the number in the unit place for odd numbers: When there is an odd digit in the unit place except 5, multiply the number by itself until you gets 1 in the unit place.

$(...1)^n$ = (… 1)

$(...3)^{4n}$ = (… 1)

$(...7)^{4n}$ = (… 1)

$(...9)^{2n}$ = (… 1) where n = 1, 2, 3, . . .

b) To find the number in the units place for even numbers: When there is an even digit in the unit place, multiply the number by itself until you gets 6 in the unit place.

$(...2)^{4n}$ = (… 6)

$(...4)^{2n}$ = (… 6)

$(...6)^n$ = (… 6)

$(...8)^{4n}$ = (… 6) where n = 1, 2, 3, . . .

c) If there is 1, 5 or 6 in the units place of the given number: If there is 1, 5 or 6 in the unit place of the given number, then after any times of its multiplication, it will have the same digit in the unit place.

$(...1)^n$ = (… 1)

$(...5)^n$ = (... 5)

$(...6)^n$ = (... 6) where n = 1, 2, 3, . . .

Solved Examples

1. On dividing 64652 by a certain number, the quotient is 101 and the remainder is 12. Find the divisor.
Sol: Here, the required number is divisor.

$$\text{Divisor} = \frac{\text{Dividend - Remainder}}{\text{Quotient}}$$

$$= \frac{64652-12}{101} = \frac{64640}{101} = 640$$

2. A number when divided by 160 leaves a remainder 52 and the quotient is 15. Find the number.
Sol: Here, the required number is dividend.
Dividend = (Divisor x Quotient) + Remainder
= (160 x 15) + 52
= 2452

3. Find the least number of 5 digits which is exactly divisible by 642.
Sol: The least number of 5 digits is 10,000.
Dividing this number by 642, the remainder is 370.
So, the required number is 10,000 + (642 - 370) = 10272.

4. Find the greatest number of 5 digits which is exactly divisible by 642.
Sol: The greatest number of 5 digits is 99,999.
Dividing this number by 642, the remainder is 489.
So, the required number is 99,999 - 489 = 99510.

5. Find the number nearest to 14800 which is exactly divisible by 245.
Sol: The remainder on dividing 14800 by 245 is 100.
So, the number required number = 14800 – 100 = 14700 which is exactly divisible by 245.

6. Find whether 577 is a prime number.
Sol: $(24)^2 = 576 < 577$ and $(25)^2 = 625 > 577$
$\therefore$ n = 25
Prime numbers less than 25 are 2, 3, 5, 7, 11, 13, 17, 19 and 23.
Since, 577 is not divisible by any of these numbers, it is a prime number.

7. How many numbers up to 531 are divisible by 15?
Sol: Divide 531 by 15.
531 = 35 x 15 + 6
The quotient is the required number and here it is 35.
So, there are 15 numbers up to 531 are divisible by 35.

8. How many numbers up to 200 are divisible by 5 and 7 together?
Sol: L.C.M. of 5 and 7 = 35.
Divide 200 by 35.
200 = 5 x 35 + 20
The quotient is the required number and here it is 5.
So, there are 5 numbers up to 200 are divisible by 35.

9. Find the number in the unit place in $(729)^{59}$.

Sol: $(729)^{59} = (729)^{58} \times 729 = (...1) \times 729 = 9$ in the unit place.

10. Find the number in the unit place in $(98)^{42}$.

Sol: $(98)^{42} = (98)^{4\times10} \times (98)^2 = (...6) \times (...4) = 4$ in the unit place.

11. Find the number in the unit place in $(636)^{36}$.

Sol: $(636)^{36} = (...6)^{36} = 6$ in the unit place.

12. Convert 0.4444…….. into a rational number.

Sol: Let x = 0.4444……..(1)

Since 1 digit (4) is repeating multiply equation 1 on both sides by 10^1.

10 x = 4.4444…….(2)

Subtract Equation 1 from 2 on both sides

10 x = 4.4444…….

- x = 0.4444…….

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9 x = 4.0000…….

$$\Rightarrow 9x = 4 \Rightarrow x = \frac{4}{9}.$$

13. Convert 5.626262…….. into a rational number.

Sol: Let x = 5.626262…….. (1)

Since 2 digits (62) is repeating multiply equation 1 on both sides by 10^2.

100 x = 562.6262…….(2)

Subtract Equation 1 from 2 on both sides

100 x = 562.6262…….

- x = 5.6262……..

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99 x = 557.0000…….

$$\Rightarrow 99x = 557 \Rightarrow x = \frac{557}{99}.$$

H.C.F. AND L.C.M. OF NUMBERS

Common Multiple: A common multiple of two or more numbers is a number which is exactly divisible by each one of them.

Eg: 32 is a common multiple of 8 and 16

Least Common Multiple (L.C.M): The least multiples among all the common multiples of given numbers is called Least Common Multiple.

Methods of finding L.C.M.

1. Method of Prime Factors

a. Resolve each given number into prime factors.
b. Take out all the factors with highest powers that occur in given numbers.
c. Find the product of these factors. This product will be L.C.M.

Eg: Find the L.C.M. of 12, 14 and 20.

Sol: 12 = 2^2 x 3

14 = 2 x 7

20 = 2^2 x 5

So, the L.C.M. = 2^2 x 3 x 5 x 7 = 420

2. Method of Division

Eg: Find the L.C.M. of 12, 15, 18 and 20.

2	12,	15,	18,	20
2	6,	15,	9,	10
3	3,	15,	9,	5
5	1,	5,	3,	5
	1,	1,	3,	1

So, the L.C.M. = 2 x 2 x 3 x 5 x 3 = 180

Common Factor: A common factor of two or more numbers is a number which divides each of them exactly.

Eg: 4 is a common factor of 8 and 12

Highest Common Factor (H.C.F): Highest common factor of two or more numbers is the greatest number that divides each one of them exactly. It is also called Greatest Common Divisor or Greatest Common Measure.

Methods of finding H.C.F.

1. Method of Prime Factors

Eg: Find the H.C.F. of 50 and 70
Sol: 50 = **2 x 5** x 5
70 = **2 x 5** x 7

Common factors are 2 and 5.
So, H.C.F. = 2 x 5 = 10

2. Method of Division

Eg: 1. Find the H.C.F. of 3332, 3724.
Sol:

$$
\begin{array}{r}
3332\overline{)3724(1} \\
\underline{3332} \\
392\overline{)3332(8} \\
\underline{3136} \\
196\overline{)392(2} \\
\underline{392} \\
X
\end{array}
$$

So, the H.C.F. of 3332, 3724 is 196.

Eg: 2. Find the H.C.F. of 10, 15 and 23.
Sol:
Step 1: First find the H.C.F. of 10 and 15. It is 5
Step 2: Then find the H.C.F. of this 5 and 23. It is 1.
So, the H.C.F. of 10, 15 and 23 is 1.

Note:

1. L.C.M. and H.C.F. of fractions

$$\text{L.C.M.} = \frac{\text{L.C.M. of the numbers in numerators}}{\text{H.C.F. of the numbers in denominato rs}}$$

$$\text{H.C.F.} = \frac{\text{H.C.F. of the numbers in numerators}}{\text{L.C.M. of the numbers in denominato rs}}$$

2. Product of two numbers = L.C.M. of two numbers x H.C.F. of two numbers.

3. To find the greatest number that will exactly divide a, b and c, simply find the H.C.F. of a, b and c.

4. To find the greatest number that will divide a, b and c leaving remainders x, y and z respectively, find the H.C.F. of $(a-x)$, $(b-y)$ and $(c-z)$.

5. To find the least number which is exactly divisible by a, b and c, simply find the L.C.M. of a, b and c.

6. To find the least number when divided by *a, b* and *c* leaving remainders x, y and z respectively, find the (L.C.M. of a, b *and* c) – k, where $k = (a-x) = (b-y) = (c-z)$.

7. To find the least number which when divided by a, b and c leaves the same remainder r in each case, find (L.C.M. of a, b and c) + r.

8. Two numbers when divided by a certain divisor give remainders r_1 and r_2. When their sum is divided by the same divisor, the remainder is r_3. Then the divisor is given by $r_1 + r_2 - r_3$.

9. A number on being divided by d_1 and d_2 successively leaves the remainders r_1 and r_2, respectively. If the number is divided by $d_1 \times d_2$, then the remainder = $(d_1 \times r_2 + r_1)$.

10. To find the greatest number that will divide x, y and z leaving the same remainder r in each case:

    Case 1: When the value of remainder r is given
    Required remainder = H.C.F. of $(x - r)$, $(y - r)$ and $(z - r)$.

    Case 2: When the value of remainder is not given
    Required remainder = H.C.F. of $|(x - y)|$, $|(y - z)|$ and $|(z - x)|$.

11. To find the n-digit greatest number which when divided by x, y and z:

    a) Leaves no remainder i.e. exactly divisible
    Step 1: Find the L.C.M. of x, y and z. Let it be L.
    Step 2: Divide the n-digit greatest number by this L. Le the remainder be R.
    Step 3: Required Remainder = (n-digit greatest number – R).

    b) Leaves remainder k in each case:
    Required Remainder = (n-digit greatest number – R) + k.

12. To find the n-digit smallest number which when divided by x, y and z:

    a) Leaves no remainder i.e. exactly divisible
    Step 1: Find the L.C.M. of x, y and z. Let it be L.
    Step 2: Divide the n-digit smallest number by this L. Le the remainder be R.
    Step 3: Required Remainder = n-digit smallest number + (L – R).

    b) Leaves remainder k in each case:
    Required Remainder = n-digit smallest number + (L – R) + k.

Solved Examples

1. Find the greatest number that will exactly divide 200 and 310.
Sol: The required number = H.C.F. of 200 and 310 = 10.

2. Find the greatest number that will divide 148, 246 and 623 leaving remainders 4, 6 and 11 respectively.
Sol:
The required number = H.C.F. of (148 – 4) (246 – 6) (623 – 11)
= H.C.F. of 144, 240, 612 = 12.

3. Find the smallest number that is exactly divisible by 45, 63 and 80.
Sol: Required number = L.C.M. of 45, 63 and 50 = 3150.

4. Find the least number which when divided by 36, 48 and 64 leaves the remainders 25, 37 and 53 respectively.
Sol: (36 – 25) = (48 – 37) = (64 – 53) = 11
Required number = (L.C.M. of 36, 48 and 64) – 11
= 576 – 11 = 565

5. Find the least number which when divided by 12, 16 and 18, will leave the remainders 7 in each case.
Sol: Required number = (L.C.M. of 12, 16 and 18) + 7
= 144 + 7 = 151

6. Find the greatest number which will divide 772 and 2778 so as to leave the remainder 5 in each case.
Sol: Required number = H.C.F. of (772 – 5) and (2778 – 5)
= H.C.F. of 767 and 2773 = 59.

7. Find the greatest number which on dividing 152, 277 and 427 leaves equal remainder.
Sol:
Required number =
H.C.F. of $|(152 - 277)|$, $|(277 - 427)|$, $|(427 - 152)|$
= H.C.F. of 125, 275 and 150 = 25.

8. Find the greatest number of 4 digits which, when divided by 12, 18, 21 and 28 leaves 4 as a remainder in each case.
Sol: L.C.M. of 12, 18, 21 and 28 = 252.
Greatest 4-digit number = 9999.

The remainder when 9999 is divided by 252 = 171
So, the required number = (9999 – 171) + 4 = 9832.

9. Find the greatest number of 4 digits which, when divided by 12, 15, 20 and 35 leaves no remainder.
Sol: L.C.M. of 12, 15, 20 and 35 = 420.
The remainder when 9999 is divided by 420 = 339
So, the required number = (9999 – 339) = 9660.

10. Find the least number of 4 digits which is divisible by 2, 4, 6 and 8.
Sol: L.C.M. of 2, 4, 6 and 8 is 24.
The least number of 4 digits = 1000
The remainder when 1000 divided by 24 = 16.
So, the required number = 1000 + (24 – 16) = 1008.

11. Find the smallest number of 4 digits when divided by 12, 18, 21 and 28 leaves remainder 5 in each case.
Sol: L.C.M. of 12, 18, 21 and 28 = 252
The least number of 4 digits = 1000
The remainder when 1000 divided by 252 = 244.
So, the required number = 1000 + (252 – 244) + 5 = 1013.

12. Two numbers when divided by a certain divisor give remainders 16 and 12 respectively. When their sum is divided by the same divisor, the remainder is 4. Find the divisor.
Sol: Required divisor = 16 +12 – 4 = 24.

13. A number on being divided by 10 and 11 successively leaves the remainders 5 and 7, respectively. Find the remainder when the same number is divided by 110.
Sol: Required remainder = 10 x 7 + 5 = 75.

14. Find the least number which when divided by 8, 10 and 15 leaves the remainders 3, 5 and 10, respectively.
Sol: Here, 8 – 3 = 10 – 5 = 15 – 10 = 5
L.C.M. of (8, 10, 15) = 120
∴ The required least number = 120 – 5 = 115.

VBODMAS

The order of various operations in exercises involving brackets and functions must be performed strictly according to the order of the letters of the word VBODMAS. Each letter of the word VBODMAS stands as follows:

V for Vinculum	:	- (bar)
B for Bracket	:	[{( )}]
O for Of	:	of
D for Division	:	÷
M for Multiplication	:	x
A for Addition	:	+
S for Subtraction	:	-

Note: There are three brackets. 1. ( ) 2. { } 3. [ ]
They are removed strictly in the order ( ), { } and [ ].

Solved Example:

1. Simplify:

$$4\frac{1}{2}-\left[3\frac{1}{5}\div 4\frac{1}{2}\,of\,5\frac{1}{3}+\left\{11-\left[3-1\frac{1}{4}-\frac{5}{8}\right]\right\}\right]$$

Sol: Given expression

$$=\frac{9}{2}-\left[\frac{16}{5}\div\frac{9}{2}of\frac{16}{3}+\left\{11-\left[3-\frac{5}{4}-\frac{5}{8}\right]\right\}\right]$$

$$=\frac{9}{2}-\left[\frac{16}{5}\div\frac{9}{2}of\frac{16}{3}+\left\{11-\left[3-\frac{5}{8}\right]\right\}\right]$$

$$=\frac{9}{2}-\left[\frac{16}{5}\div\frac{9}{2}of\frac{16}{3}+\left\{11-\frac{19}{8}\right\}\right]$$

$$=\frac{9}{2}-\left[\frac{16}{5}\div\frac{9}{2}of\frac{16}{3}+\frac{69}{8}\right]$$

$$= \frac{9}{2} - \left[\frac{16}{5} \div \frac{9}{2} \times \frac{16}{3} + \frac{69}{8}\right]$$

$$= \frac{9}{2} - \left[\frac{16}{5} \div \frac{24}{1} + \frac{69}{8}\right]$$

$$= \frac{9}{2} - \left[\frac{16}{5} \times \frac{1}{24} + \frac{69}{8}\right]$$

$$= \frac{9}{2} - \left[\frac{16}{120} + \frac{69}{8}\right]$$

$$= \frac{9}{2} - \left[\frac{16 + 1035}{120}\right]$$

$$= \frac{9}{2} - \frac{1051}{120} = \frac{540 - 1051}{120}$$

$$= -\frac{511}{120}$$

SQUARE ROOT AND CUBE ROOT

Square: A number multiplied by itself is known as the square of the given number.
Eg: square of 3 is 3 x 3 = 9

Square Root: Square root of a given number is that number which when multiplied by itself is equal to the given number. It is denoted by the symbol $\sqrt{\ }$.
Eg: square root of 16 is 4 because 4^2 = 4 x 4 = 16
Thus, $\sqrt{16}$ = 4.

Methods of finding the Square Root:

I. Prime Factorziation Method:

This method is used when the given number is a perfect square or when every prime factor of that number is repeated twice. Follow the steps as mentioned below.

1. First find the prime factors of the given number.
2. Group the factors in pairs.
3. Take one number from each pair of factors and then multiply them together. This product is the square root of the given number.

Eg: Find the square root of 225.
Sol: 225 = 5 x 5 x 3 x 3
So, √225 = 5 x 3 = 15.

II. **Method of Division:** This method is used when the number is large and the factors cannot be easily determined.

Eg: Find the square root of 180625.

```
          425
     ___________
  4 | 18  06  25
    | 16
 ---|
 82 |  206
    |  164
 ---|
845 |   4225
    |   4225
    |    x
```

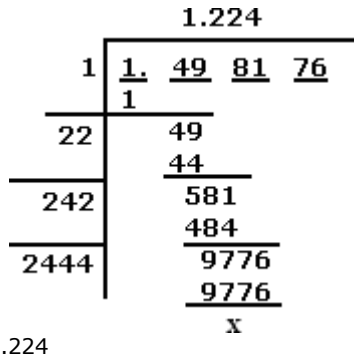
So, the square root of 180625 i.e. √180625 is 425.

Explanation:

1. First separate the digits of the number into periods of two beginning from the right. The last period may be either single digit or a pair.
2. Find a number (here it is 4) whose square may be equal or less then the first period (here it is 18).
3. Find the remainder (here it is 2) and bring down the next period (here it is 06).
4. Double the quotient (here 4) and write to the left (here 8).
5. The divisor of this stage will be equal to the above sum (here 8) with the quotient of this stage (here 2) suffixed to it (here 82).
6. Repeat this process till all the periods get exhausted.
7. The final quotient is equal to the square root of the given number (here it is 425).

Square root of a Decimal: If the given number is having decimal, separate the digits of it into periods of two to the right and left starting from the decimal point and then proceed as followed in the example.

Eg: 1. Find the square root of 1.498176.



So, √1.498176 = 1.224

Note: The square root of a decimal cannot found exactly, if it has an odd number of decimal places.

Try with finding the square root of 0.1790136

Square Root of a Fraction:

Case 1: If the denominator is a perfect square, the square root is found by taking the square root of the numerator and denominator separately.

Eg: Find the square root of $\frac{2601}{49}$.

$$\text{Sol: } \sqrt{\frac{2601}{49}} = \frac{\sqrt{2601}}{\sqrt{49}} = \frac{\sqrt{51 \times 51}}{\sqrt{7 \times 7}} = \frac{51}{7} = 7\frac{2}{7}$$

Case 2: If the denominator is not a perfect square, the fraction is converted into decimal and then square root is obtained or the denominator is made perfect square by multiplying and dividing a suitable number and then its square root can be determined.

Eg: Find the square root of $\frac{461}{8}$.

$$\text{Sol: } \sqrt{\frac{461}{8}} = \sqrt{\frac{461 \times 2}{8 \times 2}} = \frac{\sqrt{922}}{\sqrt{16}} = \frac{30.3644}{4} = 7.5911(\text{nearly})$$

Cube: Cube of a number is obtained by multiplying the number itself thrice.

Eg: 64 is the cube of 4 as 64 = 4 x 4 x 4.

Cube Root: The cube root of a number is that number which when raised to the third power produces the given number, that is the cube root of a number a is the number whose cube is a.

The cube root of a is written as $\sqrt[3]{a}$.

Methods to find Cube Root:

1. Method of Factorization:

a. First write the given number as product of prime factors.
b. Take the product of prime numbers, choosing one out of three of each type. This product gives the cube root of the given number.

Eg: Find the cube root of 9261.

Sol: 9261 = 3 x 3 x 3 x 7 x 7 x 7

So, $\sqrt[3]{9261} = 3 \times 7 = 21$.

2. Method to find Cube Roots of Exact Cubes consisting of up to 6 Digits:

Before we discuss the actual method it is better to have an overview of the following table.

Sl.No	If the cube ends in ...	then Cube root ends in	Example
1	1	1	1
2	2	8	8
3	3	7	27
4	4	4	64
5	5	5	125
6	6	6	216
7	7	3	343
8	8	2	512
9	9	9	729
10	10	0	1000

The method of finding the cube root of a number up to 6 digits which is actually a cube of some number consisting of 2 digits can be well explained with the help of the following examples.

Eg: 1. Find the cube root of 19683.

Sol: First make groups of 3 digits from the right side.

19 683 : 19 lies between 2^3 and 3^3 , so left digit is 2.
687 ends in 3, so right digit is 7. [See the table.]
Thus, the cube root of 19683 is 27.

Eg: 2. Find the cube root of 614125.

614 125 : 614125 lies between 8^3 and 9^3 , so left digit is 8.
125 ends in 5, so right digit is 5. [See the table.]
Thus, the cube root of 614125 is 85.

Exercise - 1

1. 9876543210 is divisible by
   1) 5, 9 & 11 2) 5, 9 but not by 11
   3) 9 & 11 but not by 5 4) 11 & 5 but not by 9

2. If a four digit number 1AB7 (A and B are digits) is divisible by 9 as well as by 11, then the number AB is
   1) 16 2) 28
   3) 38 4) 82

3. M and N are only two odd numbers with M > N. The largest even integer which divides $M^2 - N^2$ is
   1) 12 2) 4
   3) 6 4) 8

4. How many three-digit numbers are divisible by 6 in all?
   1) 149 2) 150
   3) 151 4) 166

5. If x and y are the two digits of the number 652xy such that this number is divisible by 80, then (x + y) is equal to?
   1) 2 2) 3
   3) 4 4) 6

6. A number of two digits is four times the sum of its digits. If 9 is added to the number, its digits are reversed. The number is?
   1) 12 2) 24
   3) 36 4) 48

7. Increasing order of the fractions $\frac{5}{6},\frac{6}{8},\frac{7}{9} and \frac{11}{13}$ will be
   1) $\frac{5}{6},\frac{6}{8},\frac{7}{9},\frac{11}{13}$ 2) $\frac{6}{8},\frac{7}{9},\frac{5}{6},\frac{11}{13}$
   3) $\frac{11}{13},\frac{5}{6},\frac{7}{9},\frac{6}{8}$ 4) $\frac{11}{13},\frac{7}{9},\frac{6}{8},\frac{5}{6}$

8. The least perfect square number which is divisible by 3, 4, 5, 6 and 8 is
   1) 1600 2) 900
   3) 2500 4) 3600

9. A boy was asked to find the value of $\frac{3}{8}$ of a sum of money. Instead of multiplying the sum by $\frac{3}{8}$, he divided it by $\frac{3}{8}$ and thus his answer exceeded by Rs.55. Find the correct answer.
   1) Rs.9 2) Rs.24
   3) Rs.64 4) Rs.1,320

10. 378 coins consist of rupee, 50 paise and 25 paise coins whose values are proportional to 13:11:7. The number of 50 paise coins will be?
   1) 132 2) 278
   3) 135 4) 136

11. The number by which 165375 should be multiplied so as to make it a perfect cube is?
   1) 2 2) 5
   3) 71 4) 7

12. $2222^{5555}+5555^{2222}$ is divisible by?
   1) 5 2) 13
   3) 7 4) 2

13. A positive integer N has exactly 12 distinct (positive) divisions including itself and 1, but only 3 distinct prime factors. If the sum of these prime factors is 20, the smallest possible value of N is
   1) 120 2) 260
   3) 308 4) None

14. The number $\frac{579632\times580001-369}{579632\times580001\times579631}$ is
   1) 1 2) 2
   3) -1 4) -2

15. If a, b and c are three positive integers such that abc + ab + ac + bc + a + b + c = 1000, then a + b + c equals to
   1) 43 2) 42
   3) 28 4) 36

16. x and y are positive integers such that 13x + 4y = 100, then (x + y) = _____
   1) 10 2) 16
   3) 14 4) 12

17. The largest value of n so that 3^n divides (251 + 261) is

1) 10 2) 11
3) 12 4) 13

18. If m and n are positive integers such that 5m + 6n = 100 then the greatest possible value mn is
1) 60 2) 70
3) 80 4) 90

19. The remainder when 1! + 2! + 3! + …….. + 100! is divided by 240 is
1) 153 2) 155
3) 165 4) 175

20. The sum of $1^2 - 2^2 + 3^2 - 4^2 + ... + 21^2 = ...$
1) 441 2) 231
3) - 231 4) – 441

21. The units digit of $1 + 9 + 9^2 + ... + 9^{2008}$ is
1) 0 2) 1
3) 9 4) 3

22. How many numbers in the list 1, 2, 3, ….., 2001 are perfect squares and also perfect cubes of whole numbers?
1) 3 2) 1
3) 4 4) more than 4

23. If 6! = a! X b! where a>1, b >1 then a + b =
1) 7 2) 6
3) 8 4) 5

24. The units digit in the product $(5+1)(5^2+1)(5^3+1).....(5^{2005}+1)$ is
1) 6 2) 5
3) 2 4) 1

25. The sum of the series $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + ... + \frac{1}{100\times 101}$ is
1) $\frac{99}{100}$ 2) $\frac{1}{100}$
3) $\frac{100}{101}$ 4) $\frac{101}{102}$

26. If f(a) = a -2 and f(a, b) = b^2 + a, then f(3, f(6)) = _____
1) 13 2) 19
3) 7 4) a^2 - 4a + 7

27. $0.\overline{023} =$ _____
1) $\frac{23}{990}$ 2) $\frac{230}{999}$
3) $\frac{23}{999}$ 4) $\frac{203}{999}$

28. Find the remainder when 2^{93} is divided by 7.
1) 2 2) 3
3) 4 4) 1

29. Find the last digit of the product 8743 x 7156 x 7567 x 8452
1) 3 2) 6
3) 4 4) 2

30. Express $0.48\overline{71}$ as a fraction.

1) $\frac{4823}{9999}$ 2) $\frac{4823}{9990}$

3) $\frac{4823}{9900}$ 4) $\frac{4832}{9900}$

31. In how many ways can 3663 be resolved into two factors?
    1) 6 2) 8
    3) 12 4) 18

32. Find the remainder when 2^{66} is divided by 65.
    1) 1 2) 17
    3) 33 4) 64

33. Find the common factor of $27^{11}+11^{11}$ and $27^{27}+11^{27}$
    1) 11 2) 38
    3) 16 4) 297

34. There are four prime numbers. The product of first three is 385 and that of the last three is 1001. The first and last numbers are ……
    1) 5, 11 2) 5, 13
    3) 7, 11 4) 7, 13

35. If the square of a number of two digits is subtracted from the square of the number formed by interchanging the digits, the largest number by which the result is always divisible by is
    1) 9 2) 11
    3) 99 4) 100

36. A number when divided by 119 leaves a remainder of 19. If it is divided by 17, it will leave a remainder ……..
    1) 16 2) 2
    3) 10 4) 7

37. The square root of $\frac{0.324\times0.081\times4.624}{1.5625\times0.0289\times72.9\times64}$
    1) 24.0 2) 0.24
    3) 0.5 4) 0.25

38. Two numbers in binary system are 110010011 and 101010101. Find their difference in decimal system.
    1) 66 2) 56
    3) 65 4) 62

PERCENTAGE

Percent: The term *per cent* means per hundred or for every hundred. The word is derived from the Latin word *per centum.*

Percentage: A fraction whose denominator is 100 is called a percentage.

Rate per cent: The numerator of the fraction is called rate per cent.

Eg: $\frac{5}{100}$ and 5 percent means the same thing i.e. 5 parts out of every hundred parts.

Basic Formulae:

1. To convert any fraction $\frac{1}{n}$ into a rate per cent, multiply it by100 and put % sign i.e.

   $\frac{1}{n}$ x 100%.

Eg: What percentage is equivalent to $\frac{3}{4}$?

Sol: $\frac{3}{4}$ x 100 = 25%

2. To convert a per cent into a fraction, drop the per cent sign and divide the number by 100.

$\frac{1}{}$

Eg: What fraction is $8\frac{1}{3}$ %?

Sol: $8\frac{1}{3}\% = \frac{25}{3} = \frac{25}{3} \times \frac{1}{100} = \frac{1}{12}$

3. x % of a given number (N) = $\frac{x}{100}$ x N

Eg: 75 % of 800 = ?

Sol: 75 % of 800 = $\frac{75}{100}$ x 800 = 600

4. If A is x % more than that of B, then B is less than that of A by $\left[\frac{x}{100+x}\times 100\right]$ %.
5. If A is x % less than that of B, then B is more than that of A by $\left[\frac{x}{100-x}\times 100\right]$ %.
6. If A is x % of C and B is y % of C, then A = $\frac{x}{y}$ x 100% of B.
7. If two numbers are respectively x % and y % more than a third number, then the first number is $\left[\frac{100+x}{100+y}\times 100\right]$ % of the second and the second number is $\left[\frac{100+y}{100+x}\times 100\right]$ % of the first.
8. If two numbers are respectively x % and y % less than a third number, then the first number is $\left[\frac{100-x}{100-y}\times 100\right]$ % of the second and the second number is $\left[\frac{100-y}{100-x}\times 100\right]$ % of the first.
9. If the price of a commodity increases by N%, then the reduction in consumption so as not to increase the expenditure is $\left[\frac{N}{100+N}\times 100\right]$ %.
10. If the price of a commodity decreases by N%, then the increase in consumption so as not to decrease the expenditure is $\left[\frac{N}{100-N}\times 100\right]$ %.
11. If a number is changed (increased/decreased) successively by x % and y % then net % change is given by $\left[x+y+\frac{xy}{100}\right]$ % which represents increase or decrease in value according as the sign is +ve or –ve.
    Note: If x and y indicates decrease in percentage, then put –ve sign before x and y else +ve sign.
12. If the population of a town (or the length of a tree) is P and its annual increase is r%, then:
    (i) Population (or length of tree) after n years = $P\left(1+\frac{r}{100}\right)^n$
    (ii) Population (or length of tree) n years ago = $\frac{P}{\left(1+\frac{r}{100}\right)^n}$.
13. If the population (or value of a machine in rupees) is P and annual decrease (or depreciation) is r%, then
    (i) Population (or value of machine) after n years = $P\left(1-\frac{r}{100}\right)^n$

(ii) Population (or value of machine) n years ago = $\frac{P}{\left(1-\frac{r}{100}\right)^n}$.

14. If a number K is increased successively by x % followed by y % and z %, then the final value of K will be

$$K\left[1+\frac{x}{100}\right]\left[1+\frac{y}{100}\right]\left[1+\frac{z}{100}\right]$$

15. In an examination, the minimum pass percentage is x%. If a student scores y marks and fails by z marks, then the maximum marks in the examination is $\frac{100(y+z)}{x}$.

16. In an examination a % and b % students respectively fail in two different subjects while c % students fail in both the subjects, then the percentage of students who pass in both the subjects will be (100 – ($a+b+c$))%.

Note: Students should solve at least two different model problems for each Formula mentioned above.

Solved Example

1. If Shashi's salary is 20% more than that of Raju, then how much percent is Raju's salary less than that of Shashi?

Sol: Here, x = 20.

Required answer = $\left[\frac{x}{100+x}\times 100\right]$%

= $\left[\frac{20}{100+20}\times 100\right]$%

= $\left[\frac{100}{6}\right]$% = $16\frac{4}{6}$% = $16\frac{2}{3}$%.

2. If Anitha's income is 30% less than that of Saritha, then how much percent is Saritha's income more than that of Saritha?

Sol: Here, x = 30.

Required answer = $\left[\frac{x}{100-x}\times 100\right]$%

= $\left[\frac{30}{100-30}\times 100\right]$%

= $\left[\frac{300}{7}\right]$% = $42\frac{6}{7}$%.

3. If A is 25% of C and B is 30% of C, then what percentage of A is B?

Sol: Here, x = 25 and y = 30

A = $\frac{x}{y}$ x 100% of B

= $\frac{25}{30}$ x 100% of B

= $\frac{25}{30}$ x 100% of B

= $\frac{500}{6}$% of B

= $83\frac{2}{6}$% of B = $83\frac{1}{3}$% of B.

4. Two numbers are respectively 25% and 50% more than a third number. What percent is the first of the second?

Sol: Here, $x = 25$ and $y = 50$

So, First number = $\left[\frac{100+x}{100+y}\times 100\right]$ % of the second

$= \left[\frac{100+25}{100+50}\times 100\right]$ % of the second

$= \frac{500}{6}$ % of the second

$= 83\frac{2}{6}\% = 83\frac{1}{3}$ % of the second.

5. Two numbers are respectively 20% and 32% less than a third number. What percent is the second of the first?

Sol: Here, $x = 20$ and $y = 32$

So, Second number = $\left[\frac{100-y}{100-x}\times 100\right]$ % of the first

$= \left[\frac{100-32}{100-20}\times 100\right]$ % of the first

= 85% of the first.

6. If the price of a commodity increases by 50%, find how much percent its consumption be reduced so as not increase the expenditure.

Sol: Reduction in consumption = $\left[\frac{N}{100+N}\times 100\right]\%$

$= \left[\frac{50}{100+50}\times 100\right]\%$

$= \left[\frac{100}{3}\right]\% = 33\frac{1}{3}\%$.

7. If the price of a commodity decreases by 50%, find how much percent its consumption be increased so as not decrease the expenditure.

Sol: Increase in consumption = $\left[\frac{N}{100-N}\times 100\right]$

$= \left[\frac{50}{100-50}\times 100\right]\%$

= 100%.

8. If the salary of Mr. Shashi is first increased by 18% and thereafter decreased by 15%, what is the net change in his salary?

Sol: Here, $x = 18$ and $y = -15$

So, the net % change in the salary = $\left[x+y+\frac{xy}{100}\right]\%$

$= \left[18-15-\frac{(18)(15)}{100}\right]\%$

$= \left[18-15-\frac{(18)(15)}{100}\right]\%$

= 0.3%.

Since the sign is +ve, there is an increase in the salary of person by 0.3%.

9. The population of a town is decreased by 20% and 40% in two successive years. What percent population is decreased after two years?

Sol: Here, x = - 20 and y = - 40

So, the net % change in population = $\left[x+y+\frac{xy}{100}\right]\%$

$$= \left[-20-40+\frac{(-20)(-40)}{100}\right]\%$$

$$= \left[-60+\frac{800}{100}\right]\%$$

= - 52%.

Since the sign is -ve, there is decrease in population after two years by 52%.

10. If the side of a square is increased by 10%, its area increased by k%. Find the value of k.

Sol: Area of square = side x side

So, net % change in area = $\left[x+y+\frac{xy}{100}\right]\%$

$$= \left[10+10+\frac{(10)(10)}{100}\right]\% \text{ [Take } x, y = 10\text{]}$$

= 21%

Hence, the area is increased by 21%.

Here, k = 21.

Exercise - 2

1. 30% of 140 =? % of 840
   1) 10 2) 5
   3) 15 4) 20

2. What percentage of Rs.400 is Rs.100?
   1) 25% 2) 30%
   3) 15% 4) 20%

3. A number exceeds its $33\frac{1}{3}$% by 180. Find the number?
   1) 540 2) 135
   3) 270 4) 300

4. If 40% of a number added to 1800, it gives the number itself. Find the number?
   1) 3000 2) 4500
   3) 2000 4) None

5. Anu's salary is 20% less than Bhanu's salary. By how much percent is the salary of Bhanu more than that of Anu?
   1) 30% 2) $33\frac{1}{3}$%
   3) 25% 4) None

6. x% of y + y% of x is equal to?
   1) x% of y 2) 2% of xy
   3) 20% of xy 4) 2% of 100xy

7. The sales of a company reduced to 20%. After how much percentage increase of the sales of the company be in original?
   1) 80% 2) 20%
   3) 100% 4) 400%

8. The salary of a worker is increased by 20% and then decreased by 20%. Find the % change in the salary
   1) 4% increase 2) 4% decrease
   3) 40% decrease 4) No Change

9. In an examination 45% of the total numbers of candidates were under 15 years age. Of these 65% were boys and there were 441 girls. Find the total number of candidates.
   1) 24,000 2) 28,000
   3) 30,000 4) None

10. The price of sugar falls by 10%. How many quintals can be bought for the same money which was sufficient to buy 18 quintals at higher price?
   1) 20 2) 10
   3) 18 4) None

11. A boy has certain number of chocolates of which he gave 13% to his brother. And 75% of the remaining to his friend, and then he has 261 left. How many did he has at first?
   1) 1300 2) 1250
   3) 610 4) 1200

12. The price of a T.V. is increased by 20% and sales are decreased by 30%. Find the percentage change in the revenue?
   1) 16% decrease 2) 10% decrease
   3) 16% increase 4) none of these

13. The side of a square is decreased by 10%. What is the percent of change in area?
   1) 20% decrease 2) 100% decrease
   3) 19% decrease 4) 21% decrease

14. A man spends 20% of his capital on raw material, 25% on building, 10% on the machinery and he was left with 4.5 lakh rupees. Find his actual capital?
   1) 1,00,000 2) 10,00,000
   3) 20,00,000 4) 15,00,000

15. A man spends 10% of his capital on raw material, 20% of the remaining on advertisement, 30% of the remaining on building, 40% of the balance on machinery and thus he was left with Rs.9072. Find his capital.
   1) 36,000 2) 40,000
   3) 45,000 4) 30,000

16. In a town 20% of the population speak English, 40% of the remaining speak Telugu, 25% of the remaining speak Tamil. 43,200 speak all other languages. Find the population of the town?
   1) 1,20,000 2) 1,00,000
   3) 1,25,000 4) 1,50,000

17. In an examination 70% of the candidates passed in English, 60% passed in Telugu and 10% failed in both. Find the pass percentage?
   1) 30% 2) 40%
   3) 50% 4) 20%

18. The population of a town is 60,000. if males are increased by 20% and females are increased by 40%. At the end of the year the population will be 78,000. Find the number of females in the town at the beginning?
   1) 30,000 2) 40,000
   3) 20,000 4) None of these

19. In an examination, 80% of the students passed in Hindi and 90% passed in Telugu, while 5% of the students failed in both the subjects. If 180 students passed in both the subjects, find the total number of students who appeared in the examination.
   1) 250 2) 240
   3) 300 4) 340

20. In an election where there are two candidates only contested. One candidate who gets 65% of the votes is elected by majority of 660 votes. Find the total number of votes.
   1) 2000 2) 2500
   3) 2200 4) 2400

21. In an election two candidates were contested. 75% of the total votes were polled. 20% of the polled votes were invalid. A person who got 40% of the valid votes was defeated by 24,000 votes. Find the number of invalid votes and polled votes.
   1) 16,000; 64,000 2) 64,000; 80,000
   3) 16,000; 1,00,000 4) 16,000; 80,000

22. A student has to obtain 35% of the total marks to pass the examination. Shyam got 30% of the total marks and failed by 15 marks. What is the maximum mark to pass?
   1) 105 2) 240
   3) 350 4) 300

23. In an examination P gets 30% of total marks and fails by 18 marks. While Q gets 40% of the total

marks and he got 12 marks more than the pass marks. Find the maximum marks and pass %?

1) 300; 35% 2) 360; 30%
3) 300; 36% 4) None of these

24. In an examination there are 2000 candidates, out of which 900 are boys. If 32% of the boys and 38% of the girls passed, find the percentage of failed candidates.

1) 67.4% 2) 64.7%
3) 65.4% 4) None of these

25. The population of a town is 1,00,000 now. If it increases at a rate of 10% p.a., find the population after 2 years?

1) 1,21,000 2) 1,20,000
3) 1,10,000 4) None of these

26. Successive discounts of 20% and 20% are equivalent to a single discount of?

1) 40% 2) 25%
3) 36% 4) None of these

27. Successive discounts of 30%, 40% and 50% are equivalent to a single discount of?

1) 90% 2) 120%
3) 80% 4) None of these

28. The marked price of a table is Rs.1,000 and two successive discounts of 30% & 20% are allowed. Find the selling price?

1) 600 2) 560
3) 660 4) None of these

29. The price of oil is increased by 25%. But a family can increase the expenditure on oil by 15% only. By what percent should that family reduce the consumption of oil?

1) 10% 2) 8%
3) 10% 4) 40%

30. In a bag containing balls, $\frac{1}{4}$ are red, $37\frac{1}{2}$ % are blue, $\frac{3}{10}$ are green and 50% of the remaining are yellow and still remaining 12 are white. What is the total numbers of balls in the bag?

1) 302 2) 340
3) 160 4) 320

31. In a group of 80 boys, 60% play chess, 75% play cricket and 55% play both. How many of them do not play any of these two games?

1) 24 2) 20
3) 18 4) 16

32. In a bookstore 25% of books are in English, 60% of the remaining are in Hindi, $33\frac{1}{3}$ % of the remaining are in Telugu and remaining 64,000 are in other languages. What is the total number of books in that bookstore?

1) 2,56,000 2) 3,20,000
3) 4,50,000 4) 6,40,000

33. Three numbers are in the ratio of 3:4:8 respectively. If the first is increased by 25%, the second is decreased by 20% and the third is unaltered, respectively their ratio will be?

1) 75:64:180 2) 75:56:160
3) 75:64:160 4) 125:64:160

34. 60% of the employees in a factory are unskilled and the rest are skilled. If 35% of the skilled employees are women and the numbers of skilled male employees are 156, then the total number of employees in that factory is?

1) 420 2) 560
3) 600 4) 720

35. In an examination, 22% students failed. If the number of passed candidates was 420 more than number of failed candidates, then the total number of candidates appeared the examination is?

1) 300 2) 324
3) 750 4) 960

Average: The average of a number of quantities of the same kind is equal to their sum divided by the number of those quantities. It is also called mean or arithmetic mean.

For example: The average of 1, 3, 5, 7 is $\frac{1+3+5+7}{4} = \frac{16}{4} = 4$

Basic Formulae:

1. $\text{Average} = \frac{\text{Sum of quantities}}{\text{Number of quantities}}$

2. Sum of quantities = Average x Number of quantities

3. $\text{Number of quantities} = \frac{\text{Sum of quantities}}{\text{Average}}$

4. If the number of quantities in two groups be a_1 and a_2 and their average is x and y, respectively, then the combined average (average of all of them put together) is given by $\frac{a_1x + a_2y}{a_1 + a_2}$.

5. If the average of a1 quantities is x and the average of a2 quantities out of them is y, the average of remaining group (rest of the quantities) is given by $\frac{a_1x - a_2y}{a_1 - a_2}$.

6. If $\bar{x}$ is the average of $x_1, x_2, \ldots\ldots, x_n$, then

a) The average of $x_1 + a, x_2 + a, \ldots\ldots, x_n + a$ is $\bar{x} + a$.

b) The average of $x_1 - a, x_2 - a, \ldots\ldots, x_n - a$ is x - a.

c) The average of $ax_1, ax_2, \ldots\ldots, ax_n$ is $a\bar{x}$, where $a \neq 0$.

d) The average of $\frac{x_1}{a}, \frac{x_2}{a}, \ldots\ldots\ldots, \frac{x_n}{a}$ is $\frac{\bar{x}}{a}$.

7) The average of n quantities is equal to x. If one of the given quantities whose value is p, is replaced by a new quantity having value q, the average becomes y, then $q = p + n(y - x)$.

8) The average of n quantities is equal to x. If a quantity is removed, the average becomes y. The value of the removed quantity is $n(x - y) + y$.

9) The average of n quantities is equal to x. If a quantity is added, the average becomes y. The value of the new quantity is $n(y - x) + y$.

10) The average of first n natural numbers is $\frac{n+1}{2}$.

11) The average of squares of natural numbers till n is $\frac{(n+1)(2n+1)}{6}$.

12) The average of cubes of natural numbers till n is $\frac{n(n+1)^2}{4}$.

13) The average of odd numbers from 1 to n is $\frac{\text{last odd number} + 1}{2}$.

14) The average of even numbers from 1 to n is $\frac{\text{last even number} + 2}{2}$.

15. **If n is odd:** The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the middle number.

16. **If n is even:** The average of n consecutive numbers, consecutive even numbers or consecutive odd numbers is always the average of the middle two numbers.

17. The average of first n consecutive even numbers is $(n + 1)$.

18. The average of first n consecutive odd numbers is n.

19. The average of squares of first n consecutive even numbers is $\frac{2(n+1)(2n+1)}{3}$.

20. The average of squares of consecutive even numbers till n is $\frac{(n+1)(n+2)}{3}$.

21. The average of squares of consecutive odd numbers till n is $\frac{n(n+2)}{3}$.

22. If the average of n consecutive numbers is m, then the difference between the smallest and the largest number is 2 $(n-1)$.

23. **Geometric Mean or Geometric Average:** It is useful in calculating averages of ratios such as average population growth rate, average percentage increase and so on.
Geometric mean of $x_1, x_2, \ldots\ldots, x_n$ is denoted by

G.M. = $\sqrt[n]{x_1, x_2, \ldots\ldots, x_n}$

24. **Harmonic Mean or Harmonic Average:** It is useful for finding average speed of a vehicle, average production per day and so on.

$$\text{H.M.} = \frac{1}{\frac{1}{n}\left[\frac{1}{x_1}+\frac{1}{x_2}\ldots\ldots\ldots+\frac{1}{x_n}\right]}$$

25. If a certain distance is covered at a speed of x kmph and the same distance is covered at a speed of y kmph, the average speed during the whole journey is $\left[\frac{2xy}{x+y}\right]$ kmph.

26. If a person or a motor car covers three equal distances at the speed of x kmph, y kmph, z kmph, respectively, then for the entire journey average speed of the person or motor car is $\left[\frac{3xyz}{xy+yz+zx}\right]$ kmph.

27. If a person covers A km at a speed of x kmph, B km at a speed of y kmph and C km at a speed of z kmph, then the average speed during the whole journey is $\left[\frac{A+B+C}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right]$ kmph.

28. If a person covers A^{th} part of the distance at x kmph, B^{th} part of the distance at y kmph and the C^{th} part at z kmph, then the average speed during the whole journey is $\left[\frac{1}{\frac{A}{x}+\frac{B}{y}+\frac{C}{z}}\right]$ kmph.

Solved Examples

1. Sunil purchased 4 toys at the rate of Rs.100 each, 6 toys at the rate of Rs.150 each and 8 toys at the rate of 200 each. What is the average cost of one toy?

Sol: Cost of 4 toys = 100 x 4 = Rs.400
Cost of 6 toys = 150 x 6 = Rs.900
Cost of 8 toys = 200 x 8 = Rs.1600
Total number of toys = 4 + 6 + 8 = 18
Average price of 1 toy = $\frac{400+900+1600}{16}$ = Rs.181.25.

2. The average mark obtained by 100 students in a competitive examination is 50. Find the total marks.

Sol: Total marks = Average marks x Number of students
= 100 x 50 = 5000.

3. The average weight of 40 students of section A of CRT class is 55 kg and that of 50

students of section B is 60 kg. Find the average weight of all the 90 students of the class.

Sol: Here, $a_1 = 40$, $a_2 = 50$ and $x = 55$, $y = 60$

Average weight = $\frac{a_1x + a_2y}{a_1 + a_2}$

$= \frac{(40)(55) + (50)(60)}{40 + 50}$

$= \frac{(40)(55) + (50)(60)}{40 + 50}$ = 57.78 kg.

4. Average salary of all the 30 employees including 5 officers of a company is Rs.750. If the average salary of the officers is Rs.1500, find the average salary of the remaining staff of the company.

Sol: Here, $a_1 = 30$, $a_2 = 5$ and $x = 750$, $y = 1500$

Average salary of the remaining staff = $\frac{a_1x - a_2y}{a_1 - a_2}$

$= \frac{30(750) - 5(1500)}{30 - 5}$

$= \frac{30(750) - 5(1500)}{30 - 5}$

= 600.

5. The average value of five numbers 7, 10, 16, 24 and 28 is 17. If 6 is added to each number, what will be the new average?

Sol: The new average = $\bar{x}$ + a = 28 + 6 = 34.

6. The average of x numbers is $4x$. If $x - 3$ is subtracted from each given number, what will be the new average?

Sol: The new average = $\bar{x}$ - a = $4x - (x - 3) = 4x + 3$.

7. The average of 8 numbers is 20. If each of the numbers is multiplied by 8, find the average of a new set of numbers.

Sol: The average of a new set of numbers = $a\bar{x}$ = 8 x 20 = 160.

8. The average weight of 20 persons is increased by 3 kg when one of them whose weight is 50 kg, is replaced by a new person. What is the weight of the new person?

Sol:

The weight of the new person, $q = p + n(y - x)$.

= 50 + 20(3) = 50 + 60 = 110 kg.

9. The average age of 24 students and the Maths teacher is 16 years. If the Maths teacher's age is excluded, the average age reduces by 1 year. What is the age of the Maths teacher?

Sol: The age of Maths teacher = $n(x - y) + y$

= 25 (16 – 15) + 15) = 40 years.

10. Find the average of first 79 natural numbers.

Sol: The required average = $\frac{n+1}{2} = \frac{79+1}{2}$ = 40.

11. Find the average of squares of the natural numbers from 1 to 47.

Sol: The required average = $\frac{(n+1)(2n+1)}{6} = \frac{(47+1)[2(47)+1]}{6}$

$= \frac{48 \times 95}{6}$ = 760.

12. Find the average of cubes of the natural numbers from 1 to 15.

Sol: The required average = $\frac{n(n+1)^2}{4} = \frac{15(15+1)^2}{4}$

$$= \frac{15\times16\times16}{4} = 960.$$

13. Find the average of odd numbers from 1 to 50.

Sol: Required average $= \frac{\text{last odd number} + 1}{2}$

$$= \frac{49+1}{2} = 25.$$

14. Find the average of even numbers from 1 to 61.

Sol: Required average $= \frac{\text{last even number} + 2}{2}$

$$= \frac{60+2}{2} = 31.$$

15. Find the average of 5 consecutive numbers 4, 5, 6, 7, 8.
Sol: The required average = middle number = 6.

16. Find the average of consecutive odd numbers 21, 23, 25, 27, 29, 31.
Sol: The required average = average of middle two numbers

$$= \frac{25+27}{2} = 26.$$

17. Find the average of first 25 consecutive even numbers.
Sol: The required average = $(n + 1) = 25 + 1 = 26$.

18. Find the average of first 30 consecutive odd numbers.
Sol: The required average = $n = 30$.

19. Find the average of squares of first 16 consecutive even numbers.

Sol: The required average $= \frac{2(n+1)(2n+1)}{3} = \frac{2(16+1)[2(16)+1]}{3}$

$$= \frac{2\times17\times33}{3} = 374.$$

Exercise:-3

1. The average of 13 papers is 40. The average of the first 7 papers is 42 and of the last seven papers is 35. Find the marks obtained in the 7th paper?
   (A) 23 (C) 19 (D) None of these
   (B) 38

2. The average age of the Indian cricket team playing the Nagpur test is 30. The average age of 5 of the players is 27 and that of another set of 5 players, totally different from the first five, is 29. If it is the captain who was not included in either of these two groups, then find the age of the captain.
   (A) 75 (C) 50 (D) Cannot be determined
   (B) 55

3. A bus goes to Ranchi from Patna at the rate of 60 km per hour. Another bus leaves Ranchi for Patna at the same times as the first bus at the rate of 70 km per hour. Find the average speed for the journeys of the two buses combined if it is known that the distance from Ranchi to Patna is 420 kilometers.
   (A) 64.615 kmph (B) 64.5 kmph (C) 63.823 kmph (D) 64.82 kmph

4. A train travels 8 km in the first quarter of an hour, 6 km in the second quarter and 40 km in the third quarter. Find the average speed of the train per hour over the entire journey.
   (A) 72 km/h (B) 18 km/h (C) 77.33 km/h (D) 78.5 km/h
5. The average weight of 6 men is 68.5 kg. If I is known that Ram and Tram weigh 60 kg each, find the average weight of the others.
   (A) 72.75 kg (C) 78 kg (D) None of these
   (B) 75 kg

6. The average score of a class of 40 students is 52. What will be the average score of the rest of the students if the average score of 10 of the students is 61.
   (A) 50 (B) 47 (C) 48 (D) 49
7. The average age of 80 students of IIM, Bangalore of the 1995 batch is 22 years. What will be the new average if we include the 20 faculty members whose average age is 37 years?

   (A) 32 years
   (B) 24 years (c)25 years(D) None of these
8. Out of the three numbers, the first is twice the second and three times the third. The average of the three numbers is 88. The smallest number is
   (A) 72 (B) 36 (C) 42 (D) 48
9. The sum of three numbers is 98. If the ratio between the first and second is 2 : 3 and that between the second and the third is 5 : 8, then the second number is
   (A) 30 (B) 20 (C) 58 (D) 48
10. The average height of 30 girls out of a class of 40 is 160 cm and that of the remaining girls is 156 cm. The average height of the whole class is
    (A) 158 cm (B) 158.5 cm (C) 159 cm (D) 157 cm
11. The average weight of 6 persons is increased by 2.5 kg when one of them whose weight is 50 kg is replaced by a new man. The weight of the new man is
    (A) 65 kg (B) 75 kg (C) 76 kg (D) 60 kg
12. The average age of A, B C and D five years ago was 45 years. By including X, the present average age of all the five is 49 years. The present age of X is
    (A) 64 years (B) 48 years (C) 45 years (D) 40 years
13. The average salary of 20 workers in an office is Rs. 1900 per month. If the manager's salary is added, the average salary becomes Rs. 2000 per month. What is the manager's annual salary?
    (A) Rs. 24, 000
    (B) Rs. 25,200 (C) Rs. 45,600 (D) None of these

14. The average weight of a class of 40 students is 40 kg. If the weight of the teacher be included, the average weight increases by 500 gm. The weight of the teacher is
    (A) 40.5 kg (B) 60 kg (C) 62 kg (D) 60.5 kg
15. In a Infosys test, a student scores 2 marks for every correct answer and loses 0.5 marks for every wrong answer. A student attempts all the 100 questions and scores 120 marks. The number of questions he answered correctly was
    (A) 50 (B) 45 (C) 60 (D) 68
16. The average of the first ten natural numbers is
    (A) 5 (B) 5.5 (C) 6.5 (D) 6
17. The average of the first ten even numbers is
    (A) 18 (B) 22 (C) 9 (D) 11
18. The average weight of a class of 30 students is 40 kg. If, however, the weight of the teacher is included, the average become 41 kg. The weight of the teacher is
    (A) 31 kg (B) 62 kg (C) 71 kg (D) 70 kg
19. 30 oranges and 75 apples were purchased for Rs. 510. If the price per apple was Rs. 2, then the average price of oranges was
    (A) Rs. 12 (B) Rs. 14 (C) Rs. 10 (D) Rs. 15
20. A batsman made an average of 40 runs in 4 innings, but in the fifth inning, he was out on zero. What is the average after fifth innings?
    (A) 32 (B) 22 (C) 38 (D) 49
21. The average weight of a school of 40 teachers is 80 kg. If, however, the weight of the principle be included, the average decreases by 1 kg. What is the weight of the principal?

    (A) 109 kg
    (B) 29 kg (C) 39 kg (D) None of these

22. The average age of Ram and Shyam is 20 years. Their average age 5 years hence will be
    (A) 25 years (B) 22 years (C) 21 years (D) 20 years

23. The average of 20 results is 30 and that of 30 more results is 20. For all the results taken together, the average is
    (A) 25 (B) 50 (C) 12 (D) 24

24. The average of 5 consecutive numbers is 18. The highest of these numbers will be

(A) 24 (B) 18 (C) 20 (D) 22

25. Three years ago, the average age of a family of 5 members was 17 years. A baby having been born, the average of the family is the same today. What is the age of the baby?
(A) 1 years (B) 2 years (C) 6 months (D) 9 months
26. Varun average daily expenditure is Rs. 10 during May, Rs. 14 during June and Rs. 15 during July. His approximate daily expenditure for the 3 months is

(A) Rs. 13 approximately (B) Rs. 12 (C) Rs.12approximately (D) Rs. 10

27. A ship sails out to a mark at the rate of 15 km per hour and sails back at the rate of 20 km/h. What is its average rate of sailing?
(A) 16.85 km (B) 17.14 km (C) 17.85 km (D) 18 km
(E)
28. The average temperature on Monday, Tuesday and Wednesday was 41 ^{0}C and on Tuesday, Wednesday and Thursday it was 40 ^{0}C. If on Thursday it was exactly 39 ^{0} C, then on Monday, the temperature was
(A) 42 ^{0}C (B) 46 ^{0}C (C) 23 ^{0}C (D) 26 ^{0}C
29. The average of 20 results is 30 out of which the first 10 results are having an average of 10. The average of the rest 10 results is
(A) 50 (B) 40 (C) 20 (D) 25

30. ten years ago, Mohan was thrice as old as Ram was but 10 years hence, he will be only twice as old. Find Mohan's present age.
a) 60 years b) 80 years c) 70 years d) 76 years

PROBLEMS ON AGES

In solving the problems related to ages, we come across three situations.

1. Age some years ago
2. Present Age
3. Age some years hence

Exercise - 4

1. The ratio of the ages of Ravi and Ramu is 3 : 2 and the sum of their ages is 20 years. The ratio of their ages after 4 years is …..
1) 4 : 5 2) 3 : 4
3) 4 : 3 4) None of these

2. At present Madhavi is 5 years older than Bindu. After 6 years Bindu will be 39 years old. What is Madhavi's present age?
1) 38 years 2) 40 years
3) 33 years 4) None of these

3. The ratio between the ages of A and B is 3 : 4. The difference between their ages 4 years ago was 6 years. Find the present age of B.
1) 25 years 2) 24 years
3) 23 years 4) None of these

4. Kavitha is 20 years younger than her father. 5 years hence, age of Kavitha will be $\frac{1}{3}$ of her father's age. Find the present age of Kavitha.
1) 15 years 2) 10 years
3) 5 years 4) None of these

5. The ratio of the ages of Ram and Yuktha is 3 : 5. After 9 years the ratio will become 3 : 4. The present age of Yuktha is ……..
1) 10 years 2) 15 years
3) 20 years 4) None of these

6. 10 years ago, the total age of father and daughter was 50 years. The ratio of their present age is 2 : 3. What is the present age of daughter?

1) 28 years 2) 27 years
3) 24 years 4) None of these

7. 7 years hence, the total age of mother and son will be 63 years. The ratio of their present ages is 5 : 2. Find the present age of the mother and the son.
1) 30, 15 years 2) 14, 35 years
3) 35, 14 years 4) None of these

8. The average age of husband and a wife was 23 years five years ago. Now the average of the husband, wife and child is 20 years. What is the present age of the child?
1) 5 years 2) 4 years
3) 6 years 4) None of these

9. Sowmya's age is $\frac{1}{6}$ of her father's age. After 10 years, her father's age will be twice of Ratan's age. If Ratan's eighth birthday was celebrated two years before, then what is the present age of Sowmya?
1) 5 years 2) 4 years
3) 6 years 4) None of these

10. 10 years ago, Koumudi's mother was four times older than her daughter. After 10 years, the mother will be twice older than her daughter. The present age of Koumudi is …….
1) 25 years 2) 15 years
3) 20 years 4) None of these

11. Vimala got married 6 years ago. Today her age is $1\frac{1}{4}$ times her age at the time of marriage. Her son's age is $\frac{1}{10}$ times her age. Her son's age is….
1) 3 years 2) 4 years
3) 5 years 4) None of these

12. In a family, a couple has a son and daughter. The age of the father is thrice that of his daughter and the age of the son is half of his mother. The wife is 9 years younger to her husband and the brother is seven year older than his sister. What is the age of the mother?
1) 50 years 2) 60 years
3) 65 years 4) None of these

13. The age of the father is twice that of the elder son. Ten years hence, the age of the father will be three times that of the younger son. If the difference of the ages of the two sons is 15 years, the age of the father is……..
1) 50 years 2) 45 years
3) 55 years 4) None of these

14. If the product of the present ages of the father and his son is 900 years and the ratio of their present ages is 25 : 9 then their present ages are respectively…
1) 50, 20 years 2) 20, 15 years
3) 50, 18 years 4) None of these

15. If 6 years are subtracted from the present age of Sudheer and the remainder is divided by 18, then the present age of his grandson Arun is obtained. If Arun is 2 years younger to Naresh whose age is 5 years, then what is the age of Sudheer?
1) 96 years 2) 60 years
3) 48 years 4) 84 years

16. The average of a class of 50 students is 14 years and the average age of another class of 30 students is 6 years. Find the average age of all the students in two classes.
1) 12 years 2) 13 years
3) 15 years 4) 11 years

17. A group of 20 girls has an average age of 12 years. Average age of first 12 from the same group is 13 years. What is the average age of other 8 girls in the group?
1) 10 years 2) 11.5 years
3) 11 years 4) 10.5 years

18. The average age of 24 boys and a class teacher of a class is 15 years. If the class teacher's age is excluded the average becomes 14. Find the age of the teacher.
1) 30 years 2) 39 years
3) 35 years 4) None of these

19. The average age of 30 boys of a class is equal to 14 years. When the age of the class teacher is

included, the average becomes 15 years. Find the age of the class teacher.

1) 45 years 2) 50 years
3) 40 years 4) None of these

20. The average weight of 8 persons increases by 1.5 kg. If a person weighing 65 kg is replaced by a new person, what could be the weight of the new person?
1) 70 kg 2) 76.5 kg
3) 77 kg 4) None of these

21. In a class, there are 20 boys whose average is decreased by 2 months. When one boy aged 18 years is replaced by a new boy, find the age of the new boy.
1) 14 years 2) 15 years
3) 15 years 8 months 4) 14 years 8 months

22. The average mark obtained by 77 candidates in a certain examination is 17. If the average mark of passed candidates is 19 and that of the failed candidates is 8, what is the number of candidates who passed the examination?
1) 36 2) 63
3) 40 4) 70

23. The average of 17 numbers is 45. The average of first 9 of those is 51 and the last 9 of those is 36. What is the ninth number?
1) 14 2) 16
3) 22 4) 18

24. The average of 15 results is 28. The average of the first 7 is 26 and that of the last 7 is 25. Find the value of 8^{th} number.
1) 36 2) 66
3) 65 4) 63

25. A batsman in his 20^{th} innings makes a score of 110 and there by increases his average by 4. What is average after 20^{th} inning?
1) 34 2) 43
3) 36 4) 30

26. A cricketer has completed 14 innings and his average is 30 runs. How many runs must he make in his next innings so as to raise his average to 32?
1) 50 2) 65
3) 60 4) 55

27. A man goes to a certain place at a speed of 15 km/hr and returns to the original place at a speed of 12 km/hr. Find the average speed during the whole journey.
1) 13 km/hr 2) $13\frac{1}{3}$ km/hr
3) $13\frac{2}{3}$ km/hr 4) $11\frac{1}{3}$ km/hr

28. A boy covers three successive kilometers at 30 km/hr, 15 km/hr and 40 km/hr. Find his average speed.
1) 24 km/hr 2) 25 km/hr
3) 36 km/hr 4) None of these

29. A man travels 18 km at 6 km/hr, 16 km at 8 km/hr and 30 km at 6 km/hr. Find the average speed in covering the whole distance.
1) 6.5 km/hr 2) 6 km/hr
3) 6.2 km/hr 4) 6.4 km/hr

30. Ravi covers 50% of the journey at 30 km/hr, 25% of the journey at 25 km/hr and the remaining at 20 km/hr. Find the average speed of the train during the whole journey.
1) $25\frac{5}{47}$ km/hr 2) $25\frac{23}{47}$ km/hr
3) $25\frac{25}{47}$ km/hr 4) None of these

31. There are 40 girls in a hostel. If the number of girls increases by 8, the expenses of the mess increased by Rs.48 per day while the average expenditure per head diminished by Rs.2. Find the original expenditure of the mess.
1) Rs.620 2) Rs.720
3) Rs.750 4) Rs.820

32. The average weight of 50 sweets is 5 gm. If the weight of the box be included, the average weight increases by 0.05 gm. What is the weight of the box?
    1) 5.75 gm
    2) 7.5 gm
    3) 7.55 gm
    4) None of these

33. Find the average of squares of first 23 consecutive even numbers.
    1) 750
    2) 754
    3) 725
    4) 752

34. Find the average of squares of first consecutive even numbers from 1 to 26.
    1) 243
    2) 236
    3) 252
    4) 235

35. Find the average of squares of odd numbers from 1 to 20.
    1) 142
    2) 136
    3) 133
    4) 144