Particle Filter Application Tobearings-Only Tracking

Chinta Mahesh, M. Prakash Reddy, S. Koteswara Rao, Kausar Jahan

Abstract:Passive tracking of a target using bearings-only measurement which is carried out in underwater scenario is most widely used. This paper considers the problem of estimating the position and velocity of a target in an underwater scenario. As bearings-only tracking is a non-linear problem, Kalman filter which is linear and traditional filter can't give correct approximations. The noise in measurement is more as underwater scenario is considered which leads to less accurate estimation. Particle filter (PF) which is a non-linear filter is considered for this problem. In PF, all particles are assigned with weights based on their likelihood computed with respect to obtained measurements. The weights assigned to the particles, after certain time period tend to equal values called sample impoverishment.The main difficulty using PF is sample degeneracy and sample impoverishment. To avoid these problems, different re-sampling techniques can be used, or PF can be combined with other filters like Extended Kalman Filter (EKF), Unscented Kalman filter (UKF), Modified Gain Bearings-only Extended Kalman filter (MGBEKF) etc. In this paper, PF with Systematic re-sampling technique and combined with MGBEKF is considered for analysing the estimation of target parameters. Evaluation of the algorithm is assessed based on the best convergence time of the solution for many scenarios using MATLAB software.

Index Terms: Bearings-only tracking, Modified Gain Bearing-only Extended KalmanFilter,Particle Filter, Signal processing, Systematic Re-sampling.

I. INTRODUCTION

Tracking is becoming an important area of research nowadays in signal as well as image processing. Target tracking plays a vital job in many practices like reconnaissance, vehicle tracking, computer surveillance rooms, image and video compression, etc. The tracking of a target using bearings- only measurement which is carried out in the underwater scenario is known as two-dimensional tracking [1]. The main problem in this tracking process is the degree of non-linearity is high. The bearings-only measurement is common in both underwater and above water scenario. Bearing is the angle made by the line of sight with respect to reference axis in a clockwise direction whereas the line of sight is the line drawing from observer to target. Target course is the angle between reference axis and heading direction of the target in clockwise direction. Similarly, observer course is the angle between the reference axis and heading direction of an observer in the

 Chinta Mahesh, Department of ECE, KoneruLakshmaiah Education Foundation, Vijayawada, India,
 M. Prakash Reddy, Department of ECE, KoneruLakshmaiah Education Foundation, Vijayawada, India**,**
 S. Koteswara Rao, Department of ECE, KoneruLakshmaiah Education Foundation, Vijayawada, India,
 Kausar Jahan, Department of ECE, KoneruLakshmaiah Education Foundation, Vijayawada, India,

clockwise direction. These are explained with the help ofFig.1 in adiagrammatic way. Tracking both position and velocity of a target is a non-linear process corresponding to parameters like course, speed and range. Here course and speed of the target are assumed to be constant. By using bearings-only measurement,target parameters like course, range and speed should be measured. The mathematical model of observer and target positions is explained in Section II. For Bearings only measurement applications it requires state estimation calculation with respect to time which includes some noisy measurement. This process considered to be non-linear and type of noise involved here as non-Gaussian. There are many methods to put forward to find the position of target [2-6]. They are Kalman filter and Extended Kalman filter (EKF) and Unscented Kalman filter (UKF).
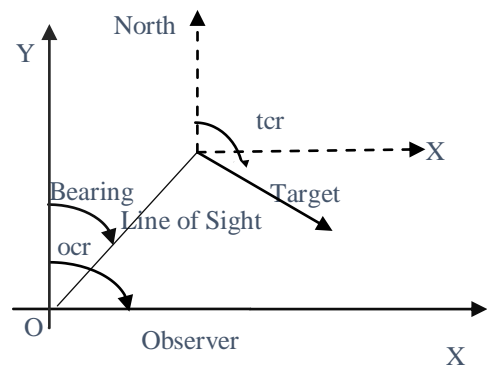

Fig. 1 Target and observer scenario

Here the traditional and optimized filter like Kalman filter does not suitable as it is a linear filter. So, choosing a filter which operates in non-linearity for bearings-only measurement is needed such as Particle filter (PF)[7-10] which is the advancement of Kalman filter is to be taken. PF is used to perform filtering for complications in the underwater situation. While considering real time situations comparing to theoretical the process of measuring parameters like course, range and speed is not linear and type of noise involved is non-Gaussian. There are many filters like Kalman and extended Kalman filter which are poor in such scenarios, PF is used along with a plain version of the algorithm Modified Gain Bearings-only EKF (MGBEKF) is explained in the analysis paper. The gain structure of the MGBEKF which is non-linear is identical to the Kalman filter as the linear system with rising stability is similar to the error dynamics of MGBEKF.With the usage ofPF, tracking of various distributions exploring the space of states with random particles will be possible [12-17].

As there are many sampling methods for PF,systematic resampling [11]is considered. Resampling is essential for PF as a few sets of particles generated during process willdominate the other set of particles, resulting in inaccurate observations. The mathematical model of PFintegrated with modified gain bearings only extended Kalman filter (PFMGBEKF) is explained in section II.

The simulations and results that are simulated usingMATLABsoftware are given in section III. Conclusion and references are shown in sections IV and V respectively.

II. MATHEMATICAL MODELING

A. Target Motion Analysis

Let (x_t, y_t) is target position with respect to observer located at the origin initially. For every 1 sec, changes in x_t and y_t are calculated and added to the previous target position for estimating the target position as follows.

$$dx_a = v_a \times sin(a_{cr}) \times a_s \qquad (1)$$
$$dy_a = v_a \times cos(a_{cr}) \times a_s \qquad (2)$$

Where= v_a is target velocity, dx_a is change in x component of a target position in one second, dy_a is changing in y component of a target position in one second, anda_{cr} is course of target with respect to true North

$$x_a = x_a + dx_a \text{ and } y_a = dy_a + dy_a \qquad (3)$$

The angle of True Bearing formula is shown below

$$B = tan^{-1} x_a/y_a$$
$$True range = R = \sqrt{(x_a - x_0)^2 + (y_a - y_0)^2}$$

Current position of target is given by

$$x_{anew} = x_{aold} + dx_a \qquad (4)$$

Finally, $Measured range = True range + noise$
$Measured bearing = True bearing + noise$

B. Observer Motion Analysis

Observer should manoeuvre in course in order to achieve Observability of the process. In this work, observer is presumed to follow S-manoeuvre. The observer position change for every second with respect to x and y co-ordinates is calculated as follows.

For $t_s = 1$ sec

$$dx_{sp} = v_{sp} \times sin(sp_{cr}) \times t_s \qquad (5)$$
$$dy_{sp} = v_{sp} \times cos(sp_{cr}) \times t_s \qquad (6)$$

where dx_{sp} is change in x-component of observer position in 1 second, dy_{sp} is change in y-component of observer position in 1 second, v_{sp} is observer velocity and sp_{cr} is observer course with respect to North. (x_{sp},y_{sp}) is observer position

$$x_{sp} = (x_{sp} + dx_{sp}) \text{ and } y_{sp} = (y_{sp} + dy_{sp})$$

The relative state vector for the next time period based on the present time state vector is calculated as

$$F_f(a1 + 1) = N(a1)F_f(a1) + \omega\Gamma(a1) \qquad (7)$$

where $N(a)$ is the system dynamics matrix calculated as

$$N(a1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \qquad (8)$$

$\Gamma(a1)$ is the process noise and ω is calculated as

$$\omega = \begin{bmatrix} t & 0 \\ 0 & t \\ t^2/2 & 0 \\ 0 & t^2/2 \end{bmatrix} \qquad (9)$$

The covariance of the process noise is calculated as

$$Q(a1) = E[(\omega\Gamma(a1))(\omega\Gamma(a1))^T]$$
$$Q(a1) = \sigma^2 \begin{bmatrix} t^2 & 0 & t^3/2 & 0 \\ 0 & t^2 & 0 & t^3/2 \\ t^3/2 & 0 & t^4/4 & 0 \\ 0 & t^3/2 & 0 & t^4/4 \end{bmatrix} \qquad (10)$$

where σ^2 represents variance in the process noise.

The measurement equation for this application has only bearing angles and the bearing angle $B(a1)$ is represented as

$$\beta_m(a1) = tan^{-1}(x_a(a1)/y_a(a1)) + Y_b \qquad (11)$$

where $Y_b$is the noise in quantification which is presumed to be following Gaussian distribution with variance σ_B^2.

C. MGBEKF

The types of noises like plant and measurement are presumed to be unconventional to oneanother. The nonlinear equation is linearized by using the Taylor series expansion. The measurement model matrix is calculated as

$$U(a1 + 1) = \begin{bmatrix} 0 \\ 0 \\ x_a(a1+1)/R^2(a1+1) \\ y_a(a1+1)/R^2(a1+1) \end{bmatrix}^T \qquad (12)$$

Since the actual values of range will not be known, the estimated range values will be used in the above equation. The predicted covariance matrix is calculated as

$$C(a1 + 1) = (N(a1 + 1)C(a1)N^T(a1 + 1)) + \omega P(a1 + 1)\omega^T \qquad (13)$$

The Kalman gain is

$$T(a1 + 1) = C(a1 + 1)U^T(a1 + 1)[\sigma_B^2 + Ua1+1Ca1+1UT(a1+1)-1 \qquad (14)$$

The updated state matrix is calculated as

$$F_f(a1 + 1) = F_f(a1 + 1) + T(a1 + 1)\left[\beta_m(a1 + 1) - Za1+1,Ffa1+1 \right. \qquad (15)$$

where $Z\left(a1 + 1, F_f(a1 + 1)\right)$ is the bearing measurement obtained from imagined estimate at time index $(a1 + 1)$. The updated covariance matrix is given in equation (16)

$$C(a1 + 1) = \left[I - T(a1 + 1)t\left(\beta_m(a1 + 1), F_f(a1 + \right.\right.$$
$$1Ca1+1I-Ta1+1t\beta ma1+1,Ffa1+1T+\sigma B2Ta1+1TTa1+1 \qquad (16)$$

where t represents the modified gain function and is calculated as follows

$$t = \begin{bmatrix} 0 & 0 & \left(\frac{cos\beta_m}{x_a sin\beta_m + y_a cos\beta_m}\right) & \left(\frac{-sin\beta_m}{x_a sin\beta_m + y_a cos\beta_m}\right) \end{bmatrix} (17)$$

D. Particle Filter

PF is originated from the concept of probability. It is a Bayesian estimator. For each step, we have to add resampling after each measurement. In this filtering algorithm, pdf of particles is computed recursively and sampled using significance of sampling technique,

which follows Monte-Carlo methodology with discrete measurements considered randomly.Initially, N particles of state vectors are randomly generated based on pdf of initialised state vector, which is presumed to be within range of knowledge. Let the system and measurement equations be

$$F_{a1+1} \; = \; s_{a1}(F_{a1}, w_{a1})$$
$$Z_{a1} \; = \; h(F_{a1}, Y_b) \quad (18)$$

At each time step, $k = 1,2 \ldots$, the particles are propagated to the next time step using system equation and measurement equations are shown

$$F_{a1,k}^{-} \; = \; s_{a1-1}(F_{a1-1}^{+}, w_{a1-1}^{k}) \qquad (19)$$

where w_{a1}, is white independent pdfwhere each w_{a1-1}^{k} noise vector is randomly created or produced on the basis of w_{a1-1} and where $w_{a1,k}^{+}$ each noise produced randomly on the basis of familiar$w_{a,k}^{-}$pdf.

Compute relative likelihood q_k of each particle $x_{a1,k}^{-}$ conditioned on measurement Z_{a1} done by evaluating pdf$p(Z_a, F_{a,k}^{-})$ (on starting point of irregular measurement equation and pdf of measurement noise. Scale the relative likelihood as follows

$$q_k = \frac{q_k}{\sum_{j=0}^{N} q_j} \qquad (20)$$

Note that the sum of all likelihoods is equal to 1.

E. Particle Filter with MGBEKF

For better accuracy we are combining PF with MGBEKF. The working of this model is shown in below flow chart.
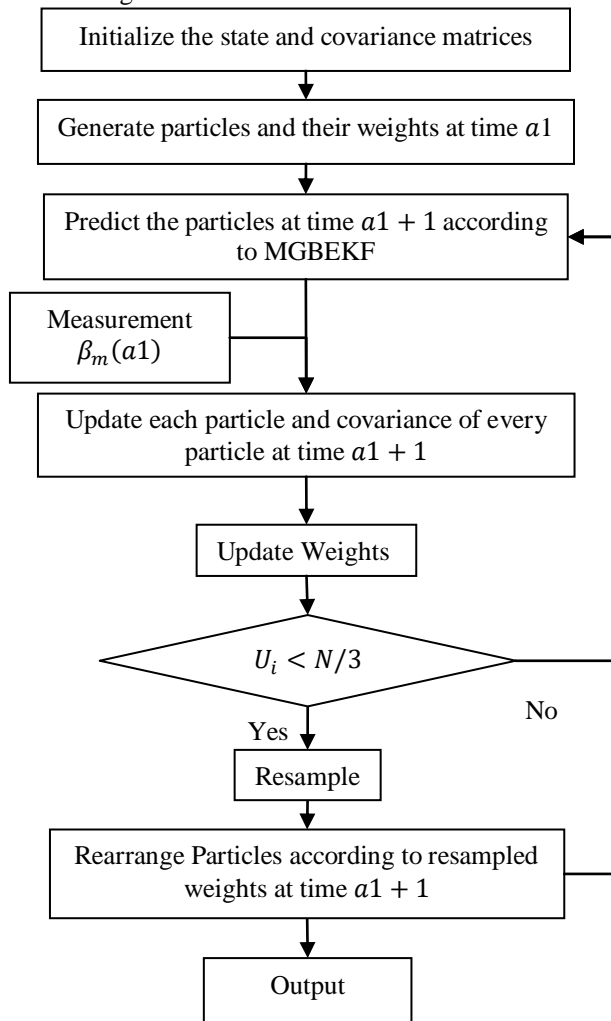

Fig 2: Flow chart for Particle Filter combined with MGBEKF

D. Systematic Resampling

Once definite sampling time period, particles with similar weights are accumulated and the particles with lesser loads become negligible. Consequently,particles with greater loads will be produced.The solution to the above problem is systematic resampling which neglects the particles with lesser weights and considering higher weights while preserving persistent number of particles. There are many types of resampling methods we use systematic resampling.Take 'N' particles and assign weights $j_i$for all these particles. The normalized weight of N particles is j_N.

$$u_k = \frac{(i-1) + \hat{u}}{N} \quad with\, \hat{u} \sim U[0,1) \qquad (21)$$

where $\hat{u}_i \sim U[0,1)$ is conventional uniform random number distribution. Allocate $S_N$copies of the particle X_N to the new distribution, where

$$S_N = the\ number\ of\ u_i \ \in \left(\sum_{a=1}^{N-1} j_a, \sum_{a=1}^{N} j_a \right]$$

Refine the set of $\emptyset(a+1)$ and $C(a+1)$ on the bases of u_i. Now, with these, we can compute any desired statistical extent of this set of weighted samples.

The algorithm of systematic resampling includes following steps. Suppose if there are Nnumber of particles, calculate the cumulative sum of weights $Q(N)$.Randomly generate weights $T(N)$. Rearrange the weights based on the condition $Q(w) > T(w)$.Whenever we get a $Q(w)$which is greater than $T(w)$ store it in the index.

III. SIMULATION AND RESULTS

This analysis paper assesses the performance of algorithm by implementing in MATLAB software. The measurements are assumed to be obtainable endlessly for each second. The observer is manoeuvring in its course. Therefore, the observer first features a course of 90^0 for 2 minutes and so turns 180^0so as to realize the primary leg in manoeuvring and features a course for 270^0. The observer requiressix minutes for full manoeuvre of 180^0 with a turning rate of 0.5^0/sec. The situationsare made believe to have totally different initial ranges, speeds and courses in more situations shown in Table1. For the target to be tracked, the observer ismade believe to be quickening with fixedspeed. The simulation results for PFMGBEKF with systematic resampling are considered. The convergence times of range, speed and course for considered scenarios are mentioned in table 2. The simulation and filtering for 100 Monte-Carlo runs are finalized for the above-mentioned scenarios using MATLAB [6]. The performance is evaluated based on the Root-Mean-Squared (RMS) error of the target parameters and the solution is obtained based on the criteria of acceptance explained as follows. The performance is assessed based on the Minimum-Mean-Squared (MMSE) inaccuracy of the target parameters and the solution is obtained based on the criteria of acceptance explained as follows. Range inaccuracy estimate<=8% of the actual range

Course inaccuracy estimate<=3°.

Speed inaccuracy estimate<=1m/s

Simulation is carried for 1200 seconds and considering 1000 particles. With systematic resampling, more accurate results are obtained.

Table1. Scenarios

Parameter	Scenario No.		
	1	2	3
Initial Range (m)	3000	3500	5000
Initial Bearing (Deg)	0	0	0
Target Speed (m/s)	12	12	8
Observer Speed (m/s)	8	8	12
Target course (deg)	135	110	135

Table2. Convergence Time (sec)

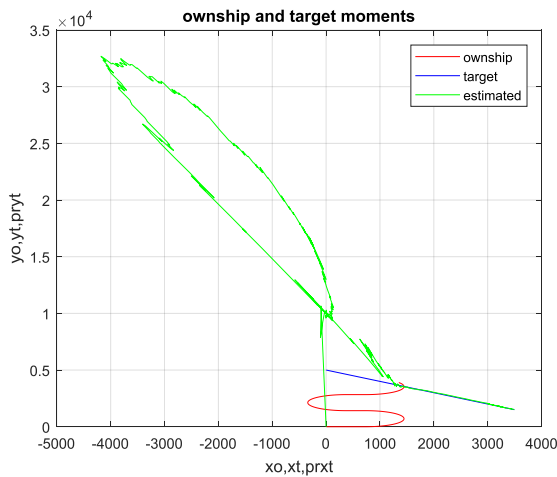| Scenario No.	Range	Speed	Course	Overall Convergence
1	257	236	356	356
2	354	209	430	430
3	432	318	496	496

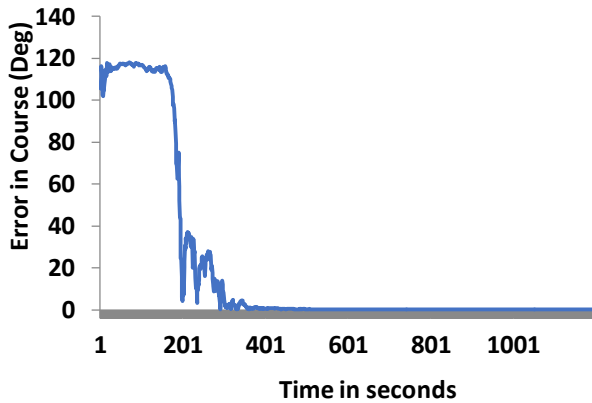**Fig 3. Target and observer movement for scenario 1**

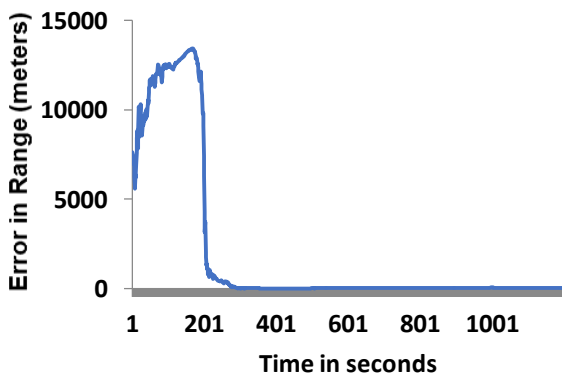
Fig 4. Error in course for scenario 1

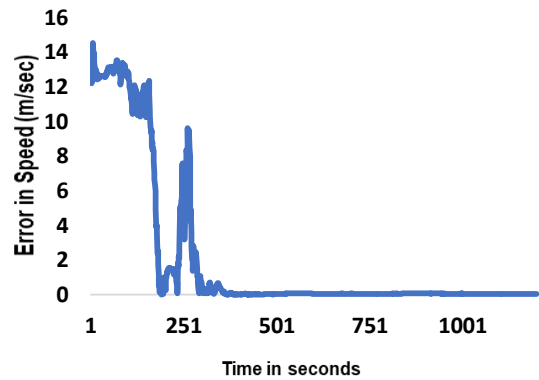**Fig 5. Error in range for scenario 1**

Fig 6 Error in speed for scenario 1

Here the graphs are drawn for course, speed and range for a time period of 1200 seconds. For scenario 1,errors in the range to be estimated, course to be estimated and the speed to be estimated of an object which is our aim obtained from the simulation are shown in figures 4-6. The convergence times for range, course and speed are 257, 236 and 356 seconds respectively and the overall convergence time of the solution is obtained at 356 seconds for scenario 1. The figure representing the target and observer movements and positions is shown in figure 3. It can be observed from the figures that the solution is obtained faster within six to seven minutes of the simulation time.

IV. CONCLUSION

In this paper analysis, finding the position and velocity of atarget with PFMGBEKF under the concept of systematic resampling is carried out,for a time period of 1200 seconds. The real-time scenarios are highly nonlinear. So, PF is considered for the optimal estimation of tracking the underwater target. For better performance of the algorithm, the PF is combined with MGBEKF. Systematic resampling technique is used for resampling the particles after each time step to avoid sample impoverishment. Different scenarios are considered to calculate the achievement of the principle. It is thus observed that the filter works efficiently, and remedy is obtained within less duration of time. Fortunately, taking required precautions during the initiation can enhance the stability and performance of the filter. Thus, we can imply that the PFMGBEKF gives the optimal estimation of the system.

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AUTHORS PROFILE

Chinta Mahesh isstudying B. Tech in Electronics and Communication Engineering in KoneruLakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India.

M. Prakash Reddy isstudying B. Tech in Electronics and Communication Engineering in KoneruLakshmaiah Education Foundation, Vaddeswaram, Guntur,Andhra Pradesh, India.

S. KoteswaraRaois former scientist "G" in NSTL, DRDO, Visakhapatnam, is currently working as Professor in KoneruLakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India. He received B Tech in 1977 at JNTU and ME in 1979, PSG college of technology, Coimbatore and Ph. D at Andhra university all in Electrical Engineering. He published several papers in International Conference & Journals in the field of signal processing. He is the fellow member of IETE.

Kausar Jahan is research scholar in KoneruLakshmaiah Education Foundation, Vaddeswaram, Guntur, Andhra Pradesh, India. She completed her B.Tech in 2009 at JNTU Kakinada and M Tech in 2015 at JNTU Kakinada all in Electronics and Communication Engineering. She is presently working as women scientist – A (WOS-A).