Application of PFMGBEKF for Bearings-only Tracking using Roughening

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Abstract: *Detection and estimation of a target in motion plays an important role in tracking. In underwater object parameters such as course, range and speed is estimated by passive sonar. In this paper particle filter is combined with modified gain bearings only extended Kalman filter (PFMGBEKF) and roughening are used. Our main assumption is that the target is moving with constant velocity. Bearing measurements are nonlinear. For such nonlinear approach sub-optimal filter is Unscented Kalman Filter (UKF). But UKF is unreliable under non-Gaussian noise environment. So, Particle Filter (PF) coupled with MGBEKF is applied and the operation analysis is based on the convergence time of the solution.. Simulations are done using MATLAB.*

Index Terms: *Bearings-only tracking, Particle filter, modified gain bearings only extended Kalman filter, Roughening.*

I. INTRODUCTION

Underwater application is used for detection and tracking of moving targets. In this application, we use sonar passive signals. Passive sonar has more detection range than active system and helps in the identification of the target and estimates the range and bearings. The passive sonar device used in the application was intended to be Omni-directional [1]. To measure the target distance we collect the information of target passive signals. Tracking of moving targets is done with the help of bearings-only-tracking (BOT) [2]. By using these bearings, target motion parameters like course, range, speed is estimated. The common problem in target motion analysis (TMA) is to estimate the velocity and position of the target. To overcome this kind of problem we use the sensors[2-3]. To get the target estimation we apply different estimation techniques to the bearings only TMA problem with varying results.. The tracking process of any targets is nonlinear and also unobservable as the observer follows the straight path and constant speed [4-6].

Kalman filter is invented by Rudolf E. Kalman who is one of the primary developers of its theory. Linear quadratic estimation (LQE) is the other name for Kalman filter

It is an algorithm which uses continuous measurements observed over time with noise and results in increasing the accuracy by estimating a joint probability distribution. The Kalman filter is commonly applied in the fields of navigation especially in aircrafts and spacecraft's [5]. It is also used in modeling the central nervous system's movement controlling. The algorithm is a two-step process. In the first step which is a predication step, the filter produces current state variables estimates and their uncertainties. Next measurements outcome is estimated and is updated using weighted average in which higher certainty estimates are given more weight. We cannot assume all the errors to be Gaussian but the filter produces a probability estimate such that all the errors are Gaussian [7-9]. A Kalman filter is used for linear estimate for linear system.

As there are a lot of nonlinear systems, Extended Kalman Filter(EKF) came into existence which is the nonlinear version of the Kalman filter. EKF mainly used techniques from calculus such as multivariate Taylor series for linearizing the model. If the system is inaccurate, particle filter is employed but is expensive. In EKF, the state transition and observation models need not be linear but may be differential functions [10]. Disadvantages of this filter are that it is not a sub-optimal estimator and if the initial estimate is incorrect or processed incorrectly, the filter diverges owing to linearization. Another issue with the EKF is that the assessed covariance matrix will in general disparage the genuine covariance matrix and hence chances of getting to be conflicting in the factual sense without the expansion of "stabilizing noise".

In this paper, Particle Filter is combined with modified gain bearings-only extended Kalman filter which is called as PFMGBEKF. The main aim of this filter is every particle is updated during the measurement time using MGBEKF and then resampled. In this paper, the PFMGBEKF is measured in the presence of Gaussian noise. Detailed mathematical modeling for this algorithm is discussed in section II. simulation and results obtained from MATLAB is presented in section III. concluded in section IV

II. MATHEMATICAL MODELING

Let the velocity of the observers are $m_{xs}(j)$, $m_{ys}(j)$ in X and Y directions and ranges are $n_{xs}(j)$, $n_{ys}(j)$. Position of the target is 'O'. Initially the target position is moving with constant speed and course. The time instant 'k' of state vector is represented as

$$v_r(j) = [\, m_{xs}(j) \quad m_{ys}(j) \quad n_{xs}(j) \quad n_{ys}(j)]^T \qquad (1)$$

Observer follows the 'S' manoeuvring and there will be change in position of observer and position change is obtained by speed and course as

$$dn_{xs}(j) = m_{xs}(j) * \sin(oc) * t \quad (2)$$
$$dn_{ys}(j) = m_{ys}(j) * \cos(oc) * t \quad (3)$$

Let $dn_{xs}(j)$ and $dn_{ys}(j)$ are change in X and Y coordinates

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and the observer course angle is given by oc and t is the time period of one second.

Let the velocity of targets are $m_{xc}(j)$ and $m_{yc}(j)$ in X and Y direction and ranges are $n_{xc}(j)$ $n_{yc}(j)$. The state vector of the target at time instant j is represented as

$$v_c(j) = [m_{xc}(j) \quad m_{yc}(j) \quad n_{xc}(j) \quad n_{yc}(j)]^T \ (4)$$

The change in target position are $dn_{xc}(j)$ and $dn_{yc}(j)$ in X and Y coordinates and ccr is target course angle and time period at one second is t.

$$dn_{xc}(j) = m_{xc}(j) * \sin(ccr) * t \ (5)$$
$$dn_{yc}(j) = m_{yc}(j) * \cos(ccr) * t \ (6)$$

The relative state vector for target with reference to observer is given as

$$u_u(j) = [m_x(j) \quad m_y(j) \quad n_x(j) \quad n_y(j)]^T (7)$$

Where $m_x(j)$, $m_y(j)$, $n_x(j)$, $n_y(j)$ are the relative complements of velocity and range in x and y directions for target.

For the next time period based on the present time state vector the relative state vector is given as

$$u_u(j + 1) = Q(j)u_u(j) + AD(j) \ (8)$$

Where $Q(j)$ represents state matrix and the value is given as

$$Q(j) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \ (9)$$

And $D(j)$ represents process noise and A is given as

$$A = \begin{bmatrix} t & 0 \\ 0 & t \\ \frac{t^2}{2} & 0 \\ 0 & \frac{t^2}{2} \end{bmatrix} (10)$$

The covariance of the process noise is given as

$$P(s) = E\left[(AD(s))(AD(s))^T\right] \ (11)$$

$$P(s) = \rho^2 \begin{bmatrix} t^2 & 0 & t^3/2 & 0 \\ 0 & t^2 & 0 & t^3/2 \\ t^3/2 & 0 & t^4/2 & 0 \\ 0 & t^3/2 & 0 & t^4/2 \end{bmatrix} (12)$$

Where ρ^2 is the variance for the process noise.

Using the bearings angle $\alpha(r)$ the measurement equation is represented as

$$\alpha(j) = \tan^{-1}\left(n_x(j)/n_y(j)\right) \ (13)$$

The degraded bearings measurement due to noise is given as

$$\alpha_x(j) = \alpha(j) + y(j) \ (14)$$

Noise in the measurement is represented by $y(j)$ and the system measurement equation is given as

$$Y(j) = g(j)u_u(j) + \gamma(j) \ (15)$$

Where $\gamma(j)$ is the measurement noise matrix and $g(j)$ represents measurement model matrix

A. MGBEKF algorithm

The plant noise and measurement noise are presumed to be independent to each other. The Taylor series expansion is used for linearizing the nonlinear equation. The measurement model matrix is calculated as

$$G(j + 1) = \begin{bmatrix} 0 \\ 0 \\ n_y(j + 1)/H^2(j + 1) \\ n_x(j + 1)/H^2(j + 1) \end{bmatrix}^T (16)$$

Since the actual values of range will not be known, the estimated range values will be used in the above equation. The predicted covariance matrix is calculated as

$$B(j + 1) = \left(Q(j + 1)B(j)Q^T(j + 1)\right) + AD(j + 1)A^T \ (17)$$

The Kalman gain is

$$Z(j + 1) = B(j + 1)G^T(j + 1)[\rho_c^2 + G(j + 1)B(j + 1)G^T(j+1)]{-1} (18)$$

The updated state matrix is calculated as

$$u_u(j + 1) = u_u(j + 1) + Z(j + 1)[\alpha_x(j + 1) - N_{j+1,uuj+1}] (19)$$

Where $N\left(j + 1, u_u(j + 1)\right)$ is the bearings measurement obtained from predicted estimate at time index $(j + 1)$. The updated covariance matrix follows the below equation

$$B(j + 1) = [I - Z(j + 1)x\left(\alpha_x(j + 1), u_u(j + 1)\right)]Q(j + 1)I - Zj+1 x a x j+1, uu j+1 T + \rho c 2 Zj+1 ZTj+1 \ (20)$$

Where x is the modified gain function and is computed as follows

$$x = \begin{bmatrix} 0 \\ 0 \\ \left(\dfrac{\cos\alpha_x}{n_x \sin\alpha_x + n_y \cos\alpha_x}\right) \\ \left(\dfrac{-\sin\alpha_m}{n_x \sin\alpha_x + n_y \cos\alpha_x}\right) \end{bmatrix}^T (21)$$

B. Particle filter

PF is a non-linear state estimator, so more computational effort is required for higher performance of the PF [10]. For repeatedly calculating the posterior probability density function of a state vector gives system and measurement equations as follows

$$c_{j+1} = f_j(c_j, w_j) \ (22)$$
$$d_j = h_l(c_j, u_j) \ (23)$$

Where $\{w_j\}$ and $\{u_j\}$ are independent white noise processes with known PDF's.

Generate randomly 'N' initial particles based on the assumed initial state PDF $p(c_o)$. These particles are termed as $c_{0,i}^+ (i = 1, ..., N)$.

For j=1, 2. . .Do the following

(a) A time propagation step is performed to get a priori particles using the process equation and PDF of the process noise $x_{k,i}^-$.

$$c_{k,i}^- = f_{j-1}\left(c_{j-1,i}^+, w_{j-1}^i\right) \ (i = 1, ..., N) \ (24)$$

w_{j-1}^i is a randomly generated noise vector.

(b) The relative likelihood q_i of each particle $c_{j,i}^-$ conditioned on the measurement d_j is computed by evaluating the PDF $p(d_j|c_{j,i}^-)$ and the PDF of the measured noise

(c) Scaling the alike likelihoods obtained in the above step as follows:

$$e_i = \frac{e_i}{\Sigma_{j=1}^N e_j} (25)$$

d) Based on the above corresponding likelihoods a set of posterior particles $c_{j,i}^+$ are generated. This is the re-sampling technique in particle filter. Roughening re-sampler is used as the re-sampling technique.

C. Particle filter coupled with modified gain bearings-only extended Kalman filter (PFMGBEKF):

Improving particle filtering is done by combining particle filter with another filter like MGBEKF. In this method at the measurement time each particle is updated using the MGBEKF, then using the measurement residual re-sampling is

performed [10]. The state $u_u(j + 1, j)$ is updated to $u_u(j + 1, j + 1)$ according to the following MGBEKF equations.

$$B(j + 1, j)_i = (Q(j + 1, j)_i B(j, j)_i Q^T(j + 1, j)_i) + AD(j + 1)A^T \quad (26)$$

$$Z(j + 1)_i = B(j + 1, j)_i G^T(j + 1)_i [\rho_c^2 + G(j + 1)_i B(j + 1, j)_i GT(j+1)i - 1 \quad (27)$$

$$u_u(j + 1, j + 1)_i = u_u(j + 1, j)_i + Y(j + 1)_i[\alpha_m(j + 1) - hj+1, uu(j+1,j)i \quad (28)$$

$$A(j + 1, j + 1)_i = [I - Z(j + 1)_i x(\alpha_x(j + 1), u_u(j + 1, j)iQ(j+1,j)i \times I - Z(j+1)ix\alpha x j+1, uu(j+1,j)iT+\rho c2Y(j+1)iZT(j+1,j)i \quad (29)$$

Where $Z(j + 1)$ represents the Kalman gain, $B(j + 1, j)i$ represents priori estimation error covariance for the ith particle and x (.) is modified gain function. x (.) is given by

$$x = \begin{bmatrix} 0 \\ 0 \\ \left(\dfrac{\cos \alpha_x}{\hat{n}_x \sin \alpha_x + \hat{n}_y \cos \alpha_x}\right) \\ \left(\dfrac{-\sin \alpha_x}{\hat{n}_x \sin \alpha_x + \hat{n}_y \cos \alpha_x}\right) \end{bmatrix}^T \quad (30)$$

D. Re-sampling:

PF mainly engages three operations, they are particle propagation, weight computation and resampling. Both particle propagation and weight computation operations are discussed in early sections of PF. In resampling, one set of particles and their weight is replaced with another set. In this, resampling step is used after the weight is normalized. After normalization, the particles with the large weight are duplicated and the particles with insignificant weights are removed. Without resampling PF will produce a depraved set of particles. Resampling methods have more superiority and various resampling methods are considered. Roughening re-sampling is the method that consists of drawing repeated samples from the original data samples. Due to replacement, the drawn samples are used by the method of re-sampling consists of repetitive cases. In roughening re-sampling method for particle filtering first we generate a random number and arrange the particles with cumulative weight > random number then adds noise to each particle according to following formula $c_{k,i}^+(x) = c_{k,i}^i(x) + \Delta x(x)$ $(c = 1, 2, ... n)$. Where $\Delta x(x)$ is noise added to particle values lies in $(0, JL(x)M^{-1/n})$, K is the scalar parameter, L is a vector containing the maximum difference between the particle elements, M is the number of particles, n is the dimension of the state space.

III. SIMULATION AND RESULTS:

MATLAB PC condition is used for the execution investigation of the molecule channel. Suspicion is made with the end goal that estimations are accessible routinely for consistently. The eyewitness is moving in its course. Along these lines, the spectator at first has the course of 90 degrees for two minutes and after that turns 180 degrees so as to accomplish the first leg in moving and has a course of 270 degrees. The eyewitness is considered to take it for four minutes for complete movement of 180 degrees. The item is thought to have distinctive starting reaches, speed and courses in various situations, which are given in the Table 1.

The article expresses vector's underlying assessment for PF joined with MGBEKF is taken as

$$u_u(0,0) = [5 \quad 5 \quad 5000 \sin\alpha_x \quad 5000 \cos\alpha_x] \quad (31)$$

accessibility of just point estimation makes troublesome for the forecast of speed parts of the item. In this way, they are each accepted as 5m/s. The item introductory position is determined dependent on the Sonar Range of the Day (SRD), which is thought to be 5000m. The underlying state covariance grid can be taken as a corner to corner lattice if the uniform circulation of the underlying state gauge is considered and given as

$$P(0,0) = diagonal \begin{bmatrix} 4s_x^2(0,0)/12 \\ 4s_y^2(0,0)/12 \\ 4g_x^2(0,0)/12 \\ 4g_y^2(0,0)/12 \end{bmatrix} \quad (32)$$

Table I Scenarios for PFMGBEKF

Scenario	Parameters				
	Range (m)	Bearings (degrees)	Target Speed (m/s)	Target Course (deg)	Observer Speed (m/s)
1.	5000	0	8	135	12
2.	3000	0	12	130	8
3.	3000	0	12	135	8
4.	3000	0	12	140	8

The simulation and filtering process for 100 particles with 1200 samples are carried out for above-mentioned scenarios in MATLAB software. The performance is analyzed based on the Root-Mean-Squared (RMS) error of the target parameters. The convergence time is obtained and listed in Table II, based on the below acceptance criteria

The acceptance criteria for this PFMGBEKF is assumed as
1. Range error estimate <= (8%)/3 of the original range
2. Course error estimate <= 1^0.
3. Speed error estimate <= 0.33m/s.

Table II Convergence in time in seconds for PFMGBEKF

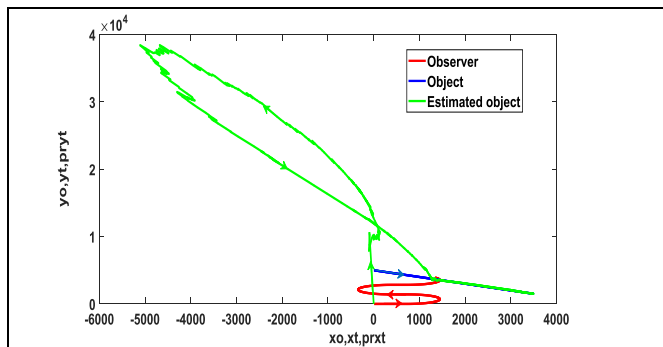| Scenario	Convergence times in seconds			
	Range	Course	Speed	Overall convergence time
1	433	663	537	663
2	292	352	324	352
3	358	370	370	370
4	266	348	324	348

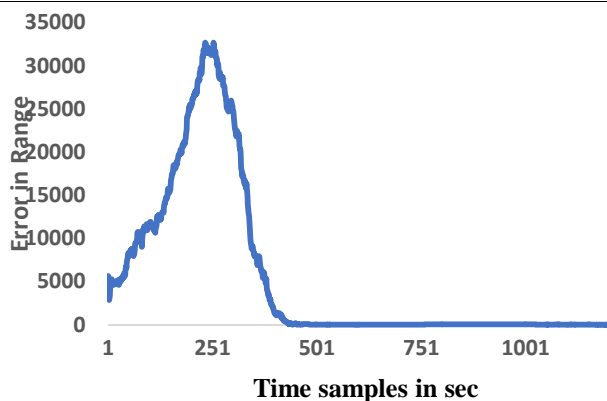**Figure 1 Observer and target movements**
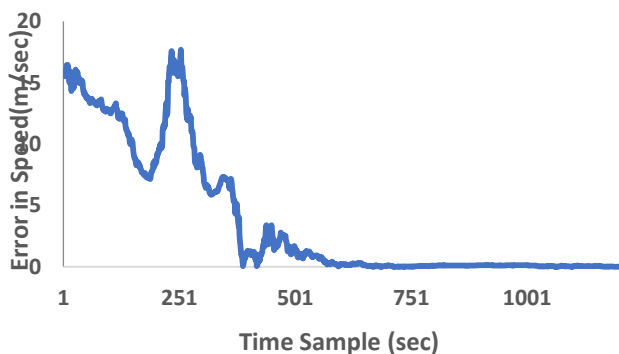

Figure 2 Error in estimates of range
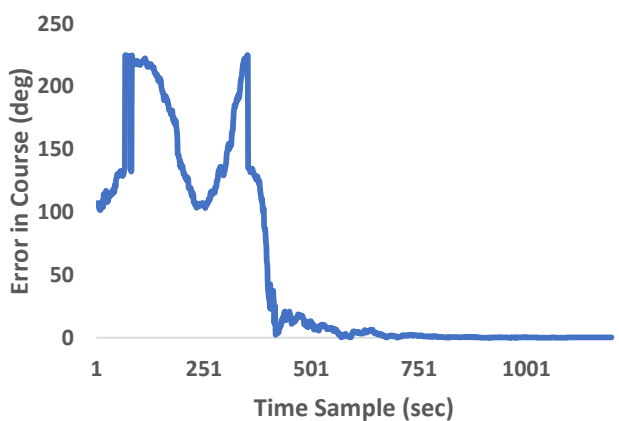

Figure 3 Error in estimates of speed

Figure 4 Error in estimates of course.

From table 2 for the scenario 1, the estimated range, course and speed of the target are 433,663,537 seconds respectively and are within the acceptable criteria for using PF with MGBEKF. The overall convergence time for scenario1 is 663 seconds.

Figure 1 contains the motion of observer, target and estimated target in 2-dimensional plane in which the observer follows maneuvering the shape of S for the validation of observability criteria from the figures 2-4 the RMS error in the estimate range, estimate course and estimate speed of the target for the PF combined with MGBEKF

IV. CONCLUSION

In this paper, analysis of PFMGBEKF for four different scenarios is done using MATLAB. Taking into account the computational effort for 100 particles is considered for simulation, which is used for better estimation of OMP. If the number of particles are increased, better convergence time can be obtained.

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