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APPLICATION OF EKF FOR UNDERWATER BEARINGS-ONLY TARGET TRACKING

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Abstract: Inactive target tracking using bearings-only measurements is a crucial issue of underwater tracking. In this paper, bearings-only measurements are used to calculate the parameters like range, course and speed components of the target in order to analyze the target motion. This is called Target Motion Analysis (TMA). TMA process is highly non-linear so the traditional, optimal linear Kalman filter will not be appropriate to use. It is presumed that the target is moving in straight line path with constant velocity, so Extended Kalman Filter (EKF) is proposed in this paper. The algorithm is simulated for several scenarios using MATLAB. Monte-Carlo runs are performed to evaluate the capability of the algorithm.

Keywords: Extended Kalman Filter; Statistical signal processing; Target tracking; Target motion analysis

1. Introduction

In underwater applications two dimensional target tracking using bearings-only measurement is often carried out. Bearing is the angle made by the line of sight from the observer to target with respect to some reference axis in the clockwise direction. A single observer platform is utilized to obtain the bearing measurements. The estimates for the target parameters of the target (range, course and speed) are acquired from these bearing measurements only. The mathematical method for obtaining these parameters is given in part A of Section II.

The process of analyzing the target motion is non-linear due to the non-linear correspondence of bearing measurements with the target state vector. Hence the Kalman filter which is an optimal linear filter [5] is not proposed. The target is presumed to travel with constant speed and constant course, so the non-linearity in the model is reduced. The non-linearity in the model is linearized by the EKF. Mathematical modeling for the filter is given in part B of Section II.

According to S. C. Nardone and V. J. Aidala one can't estimate the target parameters unless the observer makes changes its course or speed which is called maneuvering [4, 7]. Course is the angle made by the heading of the object with respect to some reference axis in the clockwise direction. If the observer makes changes in its speed then it radiates more noise and there is a risk of being tracked by the target. So the observer approaches 'S' maneuver in course. The target observer scenario is as shown in figure 1. The observer is presumed to be initially at the origin 'O' and the target at position 'T'. The observer follows 'S' maneuver for tracking the target.

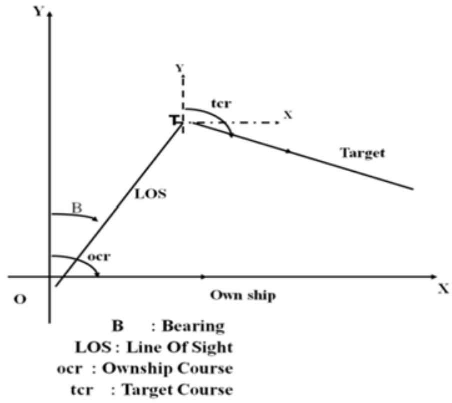

Figure. 1. Initial target- observer scenario

Section III presents the process of simulation and the different scenarios on which the simulation is done. The results are plotted as graphs and analyzed in the tables. Section IV gives the overall summary of the work done in this paper.

2. Mathematical Modeling

A. *Target Motion Analysis*

Consider the observer is at position 'O' initially and the target is moving with constant speed and course. The observer state vector at time instant 'n' [8] is given as

$S_o(n) = [v_{xo}(n) \quad v_{yo}(n) \quad r_{xo}(n) \quad r_{yo}(n)]^T$

where $v_{xo}(n), v_{yo}(n), r_{xo}(n), r_{yo}(n)$ are the velocity and range components of the observer in x and y coordinates respectively. The change in the observer position is obtained from its course and speed as

$$dr_{xo}(n) = v_{xo}(n) * \sin ocr * t$$
$$dr_{yo}(n) = v_{yo}(n) * \cos ocr * t$$

where $dr_{xo}(n), dr_{yo}(n)$ are the change in x-coordinate and y-coordinates of observer and ocr is the observer course angle and t is the time period of one second. Similarly, target state vector is given as

$$S_t(n) = [v_{xt}(n) \quad v_{yt}(n) \quad r_{xt}(n) \quad r_{yt}(n)]^T$$

where $v_{xt}(n), v_{yt}(n), r_{xt}(n), r_{yt}(n)$ are the velocity and range components of the target in x and y coordinates respectively [1]. The change in the target position is obtained from its course and speed as

$$dr_{xt}(n) = v_{xt}(n) * \sin tcr * t$$
$$dr_{yt}(n) = v_{yt}(n) * \cos tcr * t$$

where $dr_{xt}(n), dr_{yt}(n)$ are the change in x-coordinate and y-coordinates of target and tcr is the target course angle and t is the time period of one second. The relative state vector [1, 3] of the target is given as

$$S_s(n) = [v_x(n) \quad v_y(n) \quad r_x(n) \quad r_y(n)]^T \quad (1)$$

where $v_x(n), v_y(n), r_x(n), r_y(n)$ are relative components of velocity and range in x and y coordinates respectively. The relative state vector for the next time period based on the present time state vector is given as

$$S_s(n+1) = A(n)S_s(k) + \omega C(n) \quad (2)$$

where $A(n)$ is the system dynamics matrix given as

$$A(n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \quad (3)$$

and $C(n)$ is the process noise and ω is given as

$$\omega = \begin{bmatrix} t & 0 \\ 0 & t \\ t^2/2 & 0 \\ 0 & t^2/2 \end{bmatrix} \quad (4)$$

The covariance of the process noise is given as

$$Q(k) = E[(\omega C(k))(\omega C(k))^T]$$

$$Q(k) = \sigma^2 \begin{bmatrix} t^2 & 0 & t^3/2 & 0 \\ 0 & t^2 & 0 & t^3/2 \\ t^3/2 & 0 & t^4/4 & 0 \\ 0 & t^3/2 & 0 & t^4/4 \end{bmatrix} \quad (5)$$

where σ^2 is the variance of the process noise.

The measurement equation for this application has only bearing angles and the bearing angle $\beta(n)$ is given as

$$\beta(n) = \tan^{-1}\left(r_x(n)/r_y(n)\right) \quad (6)$$

The bearing measurement is always degraded with noise. So, the measured bearing is given as

$$\beta_m(n) = \beta(n) + n(n) \quad (7)$$

where $n(n)$ is the noise in the measurement. The system measurement equation is given as

$$M(n) = h(n)S_s(n) + \gamma(n) \quad (8)$$

where $h(n)$ is the measurement model matrix and $\gamma(n)$ is the measurement noise matrix.

B. *EKF Algorithm*

The EKF linearizes the non-linearities in the state and measurement equations and then performs the Kalman filtering. Here the non-linearity is considered in the measurements obtained. So the measurement model matrix is linearized using Taylor series expansion and obtained as follows

$$H(n) = [0 \quad 0 \quad \cos\beta(n)/R \quad -\sin\beta(n)/R] \quad (9)$$

where R is the range of the target from observer

$$R = \sqrt{(r_x(n))^2 + (r_y(n))^2} \qquad (10)$$

The covariance of the noise in measurement equation is given as $\emptyset(n)$ which is maximum level of Gaussian noise in bearings i.e., 0.33^0. The state vector time update equation is given as

$$S_s^-(n) = A(n-1) * S_s^+(n-1) \qquad (11)$$

The estimated state covariance matrix update equation [2] is given as

$$P^-(n) = A(n-1) * P^+(n-1) * (A(n-1))^T + Q(n-1) \qquad (12)$$

The Kalman gain [2] for the EKF is given as $G(n) = P^-(n)H^T(n)(H(n) * P^-(n)H^T(n) + \emptyset(n))^{-1}$ (13)

The measurement updates of the estimated state and estimated error covariance matrices are given respectively as follows

$$S_s^+(n) = S_s^-(n) + G(n) * Z(n) \qquad (14)$$

$$P^+(n) = \big(I - G(n) * H(n)\big) * P^-(n) * (I - G(n) * H(n))^T + G(n) * \emptyset(n) * (G(n))^T \qquad (15)$$

3. Simulation And Results

4.

The observer is maneuvering in its course. So the observer initially has a course of 90^0 for two minutes and then turns 180^0 in order to attain the first leg in maneuvering and has a course of 270^0.The observer is considered to take four minutes for complete maneuver of 180^0. The target is assumed to be having different initial ranges, speeds and courses in different scenarios, which is given in Table 1.

Table 1: Scenario for EKF algorithm

Scenarios	Parameters				
	R	B	TS	C	OS
1	3000	0	12	135	8
2	4000	0	10	110	8
3	3500	0	8	110	5

where R is the initial range in meters, B is the initial bearing in degrees, TS is the speed of the target in m/sec, C is the course of the target in degrees and OS is the speed of the observer in m/sec.

The simulation and filtering for 100 Monte-Carlo runs are carried out for the above mentioned scenarios using MATLAB [6]. The performance is evaluated based on the Root-Mean-Squared (RMS) error of the target parameters and the solution is obtained based on the criteria of acceptance explained as follows.

The acceptance criterion of the solution for the mentioned algorithm for single Monte Carlo run is:

Range error estimate<=8% of the actual range
Course error estimate<=3°.
Speed error estimate<=1m/s.

The convergence times of the solution for the three scenarios based on the above mentioned acceptance criteria for single run is tabulated in Table II.

TABLE II : Convergence time in seconds for single run

Scenario	Convergence times in seconds			
	Range	Course	Speed	Overall convergence time
1	242	252	248	252
2	232	358	163	358
3	232	426	175	426

For scenario 1, the estimated range, estimated course and estimated speed of the target obtained from simulation for single Monte-Carlo run is 242, 252 and 248 seconds respectively and the overall convergence time of the solution is obtained at 252 seconds.

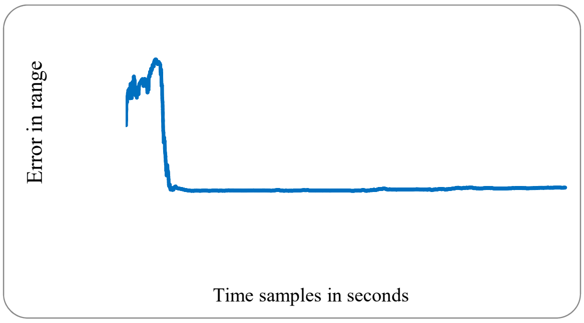

Figure 2 Error in range estimate

Figures 2 to 4 shows the error in the estimated range, course and speed of the target for single Monte-Carlo run.

The error in estimated range is reduced after the observer changes its path, until then the target path is unobservable as shown in the figure 5.

The acceptance criterion of the solution for 100 Monte-Carlo runs is assumed as

Range error estimate<= (8%)/3 of the actual range
Course error estimate<=1°.
Speed error estimate<=0.33m/s.

The convergence times of the solution for the three scenarios based on the above mentioned acceptance criteria is tabulated in Table III.

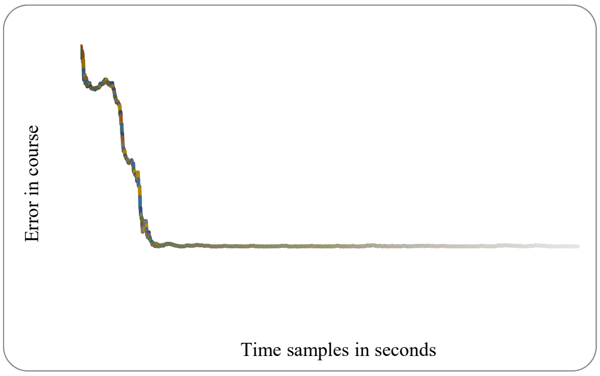

Figure 3 Error in estimated course

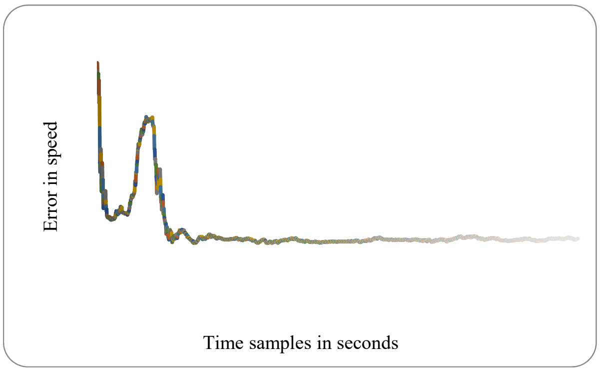

Figure 4 Error in estimated speed

Table III: Convergence time in seconds for 100 runs

Scenario	Convergence times in seconds			
	Range	Course	Speed	Overall convergence time
1	271	316	311	316
2	327	431	326	431
3	377	451	354	451

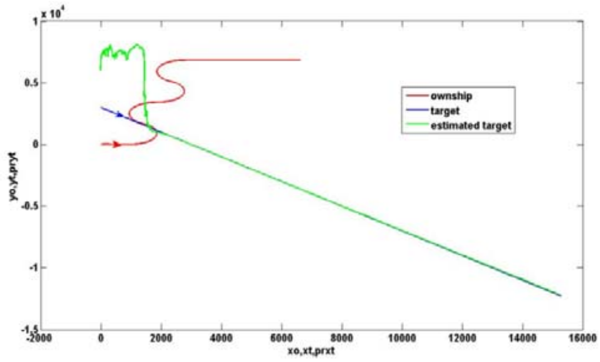

Figure.5. Observer and target movements

For scenario 1, the estimated range, estimated course and estimated speed of the target obtained from simulation for 100 Monte-Carlo runs are 271, 316 and 311 seconds respectively and the overall convergence time of the solution is obtained at 316 seconds.

Figure 5 shows the movements of the observer and target. The observer follows 'S' maneuver whereas the target moves in a straight line path. The error in estimated path is reduced after the observer changes its path, until then the target path is unobservable as shown in the figure 5.

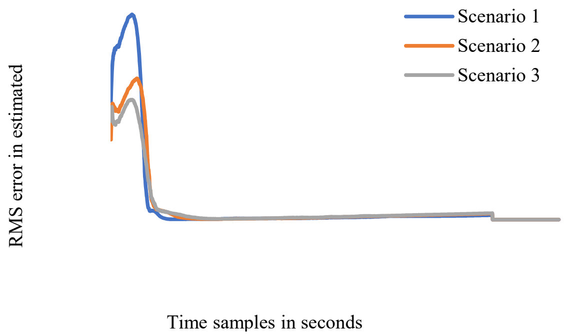

Figure.6 RMS error in range estimate

Figures 6 to 8 depicts the RMS errors in range, course and speed of the target for all the three scenarios respectively. The simulation is carried out for 100 Monte-Carlo runs so that the accuracy in estimation of the target parameters is increased.

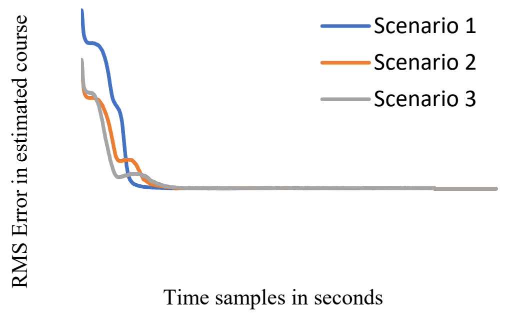

Figure.7. RMS error in course estimate

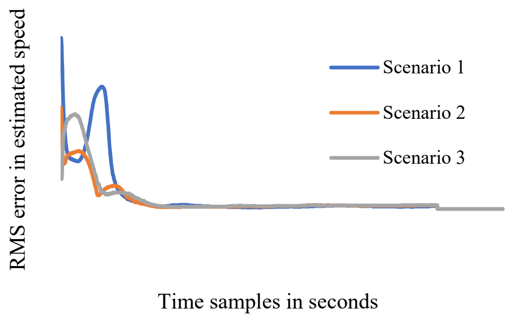

Figure.8. RMS error in speed estimate

5. Conclusion

An attempt is made to present the analysis of EKF for bearings-only target tracking. This is a crucial area of future research. Numerous scenarios were tested using Monte-Carlo simulations. But only few scenarios have been presented here which are sufficient to indicate the capability of the EKF. The filter works more efficiently only when the target becomes observable after the manoeuvring of the observer. Woefully, the EKF has no incorporated system to guarantee that anticipated estimates are used during covariance calculation. Regardless of this snag, the analysis demonstrates that significant enhancements in filter stability can be acknowledged by taking certain primary precautions with regard to initialization.

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