International Journal of Pure and Applied Mathematics
Volume 116 No. 24 2017, 369-379
ISSN: 1311-8080 (printed version); ISSN: 1314-3395 (on-line version)
url: http://www.ijpam.eu
Special Issue
ijpam.eu

MGBEKF and UKF Application to Bearings-Only Tracking

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Abstract

Inactive object tracking using bearings-only measurements is a crucial issue of underwater tracking. In this paper, bearings-only measurements are used to determine the parameters like range, course and speed components of the object in order to analyze the object motion. This is called Object Motion Analysis (OMA) and this process is highly non-linear so the Kalman filter which is a traditional, optimal linear filter will not be appropriate to relay on. It is presumed that the object is moving in an undeviating path with constant velocity. So, Unscented Kalman Filter (UKF) and Modified Gain Bearings-only Extended Kalman Filter (MGBEKF) algorithms are implemented and their performance is assessed based on their solution convergence time. The algorithms are simulated for several scenarios which are close to reality using MATLAB. Monte-Carlo runs are conducted to evaluate the capability of the algorithms.

Key Words: Stochastic signal processing, modified gain extended Kalman filter, Unscented Kalman filter, Monte-Carlo simulation.

1. Introduction

Two dimensional tracking of objects with bearings-only measurements is often carried out in underwater applications [1]. Bearing is the angle made by the line of sight from the observer to object with respect to some reference axis in the clockwise direction. A single observer platform is utilized to obtain the bearing measurements. The estimates for the object parameters (range, course and speed) are acquired from these bearing measurements only. The mathematical method for obtaining these parameters is provided in Section 2.1.

Analyzing the object's motion is a non-linear process as the correspondence of bearing measurements with the object state vector is nonlinear. Hence the Kalman filter which is an optimal linear filter [1-8] is not proposed. The object is presumed to travel with a constant course and speed, so the non-linearity of the model will be reduced. The plant noise considered is white Gaussian noise generated due to disturbance in the velocity of object.

The non-linearity in the model is linearized by the EKF. The practicality of Speyer's modified gain extended Kalman filter (MGEKF) [3] along with the simpler version of algorithm introduced by Galkowski [4] are considered and the algorithm Modified Gain Bearings-only EKF (MGBEKF) is proposed in this paper. The algorithm for MGBEKF is given in section 2.2.

Another algorithm that is employed for comparison is Unscented Kalman Filter (UKF). Unscented Transformation (UT) is the basis for UFK algorithm [5-7]. In UT, a fixed number of sigma points are chosen deterministically with some mean and covariance. From each sigma point the moments of the transformed variable are estimated by propagation of the sigma points through a nonlinear function. The efficiency to capture the higher order moments obtained during the nonlinear transform keeps UT ahead of Taylor series based approximation. The mathematical modeling for UKF is explained in section 2.3.

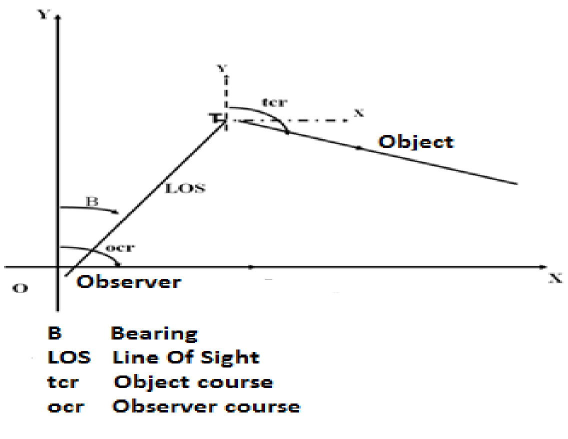

Figure 1: Initial Object- Observer Scenario

According to S. C. Nardone and V. J. Aidala one can't estimate the object parameters unless the observer makes changes its course or speed which is called maneuvering [10]. Course is the angle made by the heading of the object with respect to some reference axis in the clockwise direction. If the observer makes changes in its speed then it radiates more noise and there is a risk of being tracked by the object. So the observer approaches 'S' maneuver in course. The object observer scenario is as shown in figure 1. The observer is presumed to be initially at position 'O' and the object at position 'T'. The observer follows 'S' maneuver for tracking the object.

Performance of the two algorithms is assessed based on the best convergence time of the solution for the three scenarios given in Table 1. Section 3 presents the process of simulation and the different scenarios on which the simulation is done. The results are plotted as graphs and analyzed in the tables. Section 4 gives the overall summary of the work done in this paper.

2. Mathematical Modelling

Object Motion Analysis

Consider the observer is at position 'O' initially and the object is moving with constant speed and course. The state vector at time instant 'n' of the observer [8] is represented as

$$S_o(n) = [v_{xo}(n) \quad v_{yo}(n) \quad r_{xo}(n) \quad r_{yo}(n)]^T$$

where $v_{xo}(n)$, $v_{yo}(n)$, $r_{xo}(n)$, $r_{yo}(n)$ are the velocity and range components of the observer in x and y coordinates respectively. The change in the observer position is obtained from its course and speed as

$$dr_{xo}(n) = v_{xo}(n) * \sin ocr * t$$
$$dr_{yo}(n) = v_{yo}(n) * \cos ocr * t$$

where $dr_{xo}(n), dr_{yo}(n)$ are the change in x-coordinate and y-coordinates of observer and ocr is the observer course angle and t is the time period of one second. Similarly, object state vector is represented as

$$S_t(n) = [v_{xt}(n) \quad v_{yt}(n) \quad r_{xt}(n) \quad r_{yt}(n)]^T$$

where $v_{xt}(n)$, $v_{yt}(n)$, $r_{xt}(n)$, $r_{yt}(n)$ are the velocity and range components of the object in x and y coordinates respectively [1]. The change in the object position is obtained from its course and speed as

$$dr_{xt}(n) = v_{xt}(n) * \sin tcr * t$$
$$dr_{yt}(n) = v_{yt}(n) * \cos tcr * t$$

where $dr_{xt}(n), dr_{yt}(n)$ are the change in x-coordinate and y-coordinates of object and tcr is the object course angle and t is the time period of one second.The relative state vector [1, 3] of the object is represented as

$$S_s(n) = [v_x(n) \quad v_y(n) \quad r_x(n) \quad r_y(n)]^T \qquad (1)$$

where $v_x(n)$, $v_y(n)$, $r_x(n)$, $r_y(n)$ are relative components of velocity and range in x and y coordinates respectively. The relative state vector for the next time period based on the present time state vector is calculated as

$$S_s(n+1) = A(n)S_s(n) + b(n+1) + \omega C(n) \tag{2}$$

where $A(n)$ is the system dynamics matrix calculated as

$$A(n) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ t & 0 & 1 & 0 \\ 0 & t & 0 & 1 \end{bmatrix} \tag{3}$$

$C(n)$ is the process noise and ω is calculated as

$$\omega = \begin{bmatrix} t & 0 \\ 0 & t \\ t^2/2 & 0 \\ 0 & t^2/2 \end{bmatrix} \tag{4}$$

$b(n)$ is a deterministic matrix and is calculated as

$$b(n+1) = \begin{bmatrix} 0 \\ 0 \\ -\left(r_{xo}(n+1) - r_{xo}(n)\right) \\ -\left(r_{yo}(n+1) - r_{yo}(n)\right) \end{bmatrix}^T$$

The covariance of the process noise is calculated as

$$Q(n) = E[(\omega C(n))(\omega C(n))^T]$$

$$Q(n) = \sigma^2 \begin{bmatrix} t^2 & 0 & t^3/2 & 0 \\ 0 & t^2 & 0 & t^3/2 \\ t^3/2 & 0 & t^4/4 & 0 \\ 0 & t^3/2 & 0 & t^4/4 \end{bmatrix} \tag{5}$$

where σ^2 represents variance in the process noise.

The measurement equation for this application has only bearing angles and the bearing angle $\beta(n)$ is represented as

$$\beta_m(n) = \tan^{-1}\left(r_x(n)/r_y(n)\right) + \Upsilon_b \tag{6}$$

where Υ_b is the noise in measurement which is assumed to be following Gaussian distribution with variance σ_B^2.

MGBEKF Algorithm

The plant noise and measurement noise are presumed to be independent to each other. The nonlinear equation (6) is linearized by using the Taylor series expansion. The measurement model matrix is calculated as

$$H(n+1) = \begin{bmatrix} 0 \\ 0 \\ r_y(n+1)/R^2(n+1) \\ r_x(n+1)/R^2(n+1) \end{bmatrix}^T \qquad (7)$$

Since the actual values of range will not be known, the estimated range values will be used in the above equation. The predicted covariance matrix is calculated as

$$P(n+1) = (A(n+1)P(n)A^T(n+1)) + \omega C(\mathrm{n}+1)\omega^{\mathrm{T}} \qquad (8)$$

The Kalman gain is

$$G(n+1) = P(n+1)H^T(n+1)[\sigma_B^2 + H(n+1)P(n+1)H^T(n+1)]^{-1} \qquad (9)$$

The updated state matrix is calculated as

$$S_s(n+1) = S_s(n+1) + G(n+1)\left[\beta_m(n+1) - M\big(n+1, S_s(n+1)\big)\right] \quad (10)$$

where $M\big(n+1, S_s(n+1)\big)$ is the bearing measurement obtained from predicted estimate at time index $(n+1)$. The updated covariance matrix is given in equation (11)

$$P(n+1) = \left[I - G(n+1)g\big(\beta_m(n+1), S_s(n+1)\big)\right] * P(n+1) * \left[I - G(n+1)g\big(\beta_m(n+1), S_s(n+1)\big)\right]^T + \sigma_B^2 G(n+1)G^T(n+1) \quad (11)$$

where g represents the modified gain function and is calculated as follows [12]

$$g = \left[0 \quad 0 \quad \left(\frac{\cos\beta_m}{r_x \sin\beta_m + r_y \cos\beta_m}\right) \quad \left(\frac{-\sin\beta_m}{r_x \sin\beta_m + r_y \cos\beta_m}\right)\right] \quad (12)$$

UKF Algorithm

UKF is a straight forward add-on of the UT to the recursive estimation. In UKF, the concatenation of the original states and noise variables are delineated as the state random variables. The sigma point selection method of UT is implemented to the delineated state random variables to calculate the corresponding matrix of sigma points.

A random variable x is considered to be propagating through a nonlinear function $y = U(x)$. Consider $\bar{x}$ as the mean of x and P_x as the covariance of x. The statistics of y are calculated by considering a matrix χ of sigma vectors χ_i with i having a maximum value of $2L_1 + 1$(where L_1 is the dimension of x). The sigma vectors χ_i are assigned with corresponding weights W_i. The matrix χ is formed by using the following equations [13]:

$$\chi_0 = \bar{x}$$

$$\chi_i = \bar{x} + \left(\sqrt{(L_1+\lambda) + P_x}\right)_i \qquad i = 1,2,\dots\dots, L_1$$

$$\chi_i = \bar{x} - \left(\sqrt{(L_1+\lambda) + P_x}\right)_{i-L_1} \qquad i = L_1+1,\dots\dots,2L_1$$

$$W_0^{(m)} = \lambda/(L_1+\lambda) \qquad (13)$$

$$W_0^{(c)} = \lambda \,/\, ((L_1 + \lambda) + (1 - \vartheta^2 + \xi))$$
$$W_i^{(m)} = W_i^{(c)} = 1/\big(2(L_1 + \lambda)\big) \qquad i = 1,2, \dots ,2L_1$$

where $\lambda = \vartheta^2(L_1 + \alpha) - L_1$ is a scaling parameter. ϑ is set to a small positive value (e.g., 1e-3) that determines how the sigma points are spread around the mean. α, which is set to zero, is a secondary scaling parameter and ξ incorporates prior knowledge of the distribution of x (for Gaussian distribution, $\xi = 2$ is optimal). $\left(\sqrt{(L_1 + \lambda) + P_x}\right)_i$ represents the i^{th} row of the matrix square root. $W_0^{(m)}$, $W_0^{(c)}$, $W^{(m)}$ and $W^{(c)}$ represents the weights of initialized object state vector, state covariance matrix, state sigma point vector and state sigma point covariance matrix respectively. The nonlinear function used for propagating these sigma vectors is represented as

$$y_i = U(\chi_i) \qquad i = 1,2, \dots .,2L_1 \tag{14}$$

The weighted mean and covariance of posterior sigma points are utilized to estimate the mean and covariance of x [13].

The UKF implementation steps are as follows:

(a) Let L_1 be the dimension of object state vector. $(2L_1 + 1)$ state vectors are calculated from the initial points using sigma points

$$S(n) = \begin{bmatrix} S_s(n) \\ S_s(n) + \sqrt{(L_1 + \lambda) + P(n)} \\ S_s(n) - \sqrt{(L_1 + \lambda) + P(n)} \end{bmatrix}^T \tag{15}$$

(b) Based on the process model equation (2), transform the sigma points.

(c) The predicted state estimate at time $(n + 1)$ with n measurements is calculated as

$$S_s(n + 1) = \textstyle\sum_{i=0}^{2L_1} W_i^{(m)} S_s\big(i, (n + 1)\big) \tag{16}$$

(d)The predicted covariance matrix, assuming additive and independent process noise, is calculated as

$$P(n + 1) = \textstyle\sum_{i=0}^{2L_1} W_i^{(c)} \big[S_s\big(i, (n + 1)\big) - S_s(n + 1)\big] \times \big[S_s\big(i, (n + 1)\big) - S_s(n + 1)\big]^T + Q(n) \tag{17}$$

(e) The sigma points are updated using the predicted mean and predicted covariance as follows

$$S(n + 1) = \begin{bmatrix} S_s(n + 1) \\ S_s(n + 1) + \sqrt{(L_1 + \lambda) + P(n + 1)} \\ S_s(n + 1) - \sqrt{(L_1 + \lambda) + P(n + 1)} \end{bmatrix}^T \tag{18}$$

(f) Based on the measurement model given in equation (16), transform the predicted sigma points.

(g)Predicted measurement matrix is calculated as

$$\widehat{M}(n + 1) = \textstyle\sum_{i=0}^{2L_1} W_i^{(m)} Y(n + 1) \tag{19}$$

where $Y(n + 1) = h(S_s(n + 1))$ (20)

(h)The innovation covariance matrix is calculated as

$$P_{yy} = \sum_{i=0}^{2L_1} W_i^{(c)} \left[Y\big(i,(n+1)\big) - \widehat{M}(n+1)\right] \left[Y\big(i,(n+1)\big) - \widehat{M}(n+1)\right]^T + \sigma_B^2(n) \quad (21)$$

(i) The cross covariance matrix is calculated as

$$P_{xy} = \sum_{i=0}^{2L_1} W_i^{(c)} \left[S_s\big(i,(n+1)\big) - S_s(n+1)\right] \left[S_s\big(i,(n+1)\big) - S_s(n+1)\right]^T \quad (22)$$

Kalman gain is calculated as

$$G(n+1) = P_{xy} P_{yy}^{-1} \quad (23)$$

(j) The estimated state is calculated as

$$S(n+1) = S(n+1) + G(n+1)\left(\widehat{M}(n+1) - \widehat{M}(n+1)\right) \quad (24)$$

where $M(n+1)$ is measurement vector matrix.

(k) Error covariance matrix estimation is calculated as

$$P(n+1) = P(n+1) - G(n+1) P_{yy} G^T(n+1) \quad (25)$$

3. Simulation and Results

This research paper assesses the performance of both algorithms by implementing in MATLAB PC environment. The measurements are assumed to be available continuously for every second. The observer is maneuvering in its course. So the observer initially has a course of 90^0 for two minutes and then turns 180^0 in order to attain the first leg in maneuvering and has a course of 270^0.The observer is considered to take four minutes for complete maneuver of 180^0. The object is assumed to be having different initial ranges, speeds and courses in different scenarios, which is given in Table 1.

The object state vector's initial estimate for implementation of both algorithms is taken as

$$S_s(0,0) = [5 \quad 5 \quad 5000 \sin\beta_m \quad 5000 \cos\beta_m]$$

The prediction of velocity components of the object is difficult as only angle measurements are available. So they are each assumed as 5m/s. The object's initial position is calculated based on the Sonar Range of the Day (SRD), which is assumed to be 5000m. The initial state covariance matrix can be taken as a diagonal matrix if the uniform distribution of initial state estimate is considered and is given as

$$P(0,0) = diagonal \begin{bmatrix} 4v_x^2(0,0)/12 \\ 4v_y^2(0,0)/12 \\ 4r_x^2(0,0)/12 \\ 4r_y^2(0,0)/12 \end{bmatrix}$$

The simulation and filtering for 100 Monte-Carlo runs are performed for the above mentioned scenarios using MATLAB [6] for both MGBEKF and UKF algorithms. The performance is assessed based on the Root-Mean-Squared (RMS) error of the object parameters and the solution is obtained based on the criteria of acceptance explained as follows.

Range error estimate<=2.66% of the actual range
Course error estimate<=1^{o}.
Speed error estimate<=0.33m/s.

The convergence time of the solutions for the three scenarios based on the above mentioned acceptance criteria for 100 runs are tabulated in Table 2 for UKF algorithm and Table 3 for MGBEKF algorithm.

For scenario 1, the estimated range, estimated course and estimated speed of the object within the acceptance criteria are obtained from simulation at 376, 367 and 400 seconds respectively using UKF algorithm and the overall convergence time of the solution is obtained at 400 seconds. For the same scenario, the parameters of the object within the acceptance criteria are obtained from simulation at 256, 280 and 262 seconds respectively using MGBEKF algorithm and the overall convergence time of the solution is obtained at 280 seconds.

It can be observed from the tables 2 and 3 that the solution convergence of MGBEKF algorithm is faster when compared to that of UKF algorithm for all the scenarios. Though the computational complexity of MGBEKF is a little higher than that of UKF, the convergence of the solution plays a key role in realistic scenarios.

Figures 3-5 shows the comparison of RMS errors in estimates of range, estimates of course and estimates of speed of the object for both MGBEKF and UKF algorithms. It can be observed from the figures that MGBEKF algorithm attain low RMS error values faster than the UKF algorithm which leads to faster convergence of the solution.

Table 1: Scenarios for the Given Algorithms

Scenarios	Initial Range (m)	Initial Bearing (deg)	Object Speed (m/s)	Observer speed (m/s)	Object course (deg)
1	3000	0	12	8	135
2	3500	0	12	10	110
3	4500	0	8	5	135

Table 2: Convergence Time in Seconds for 100 Runs

Scenario	UKF			
	Range	Course	Speed	Total scenario
1	376	367	400	400
2	412	448	438	448
3	421	455	463	455

Table 3: Convergence Time in Seconds for 100 Runs

Scenario	MGBEKF			
	Range	Course	Speed	Total scenario
1	256	280	262	280
2	297	355	302	355
3	341	419	373	419

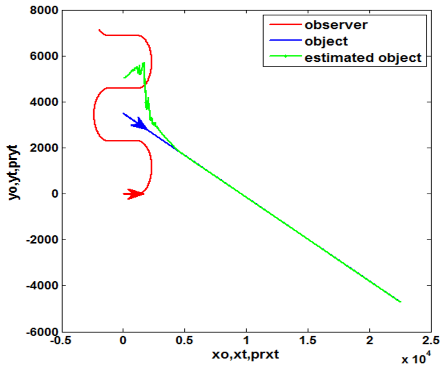

Figure 2: Observer and Object Movement

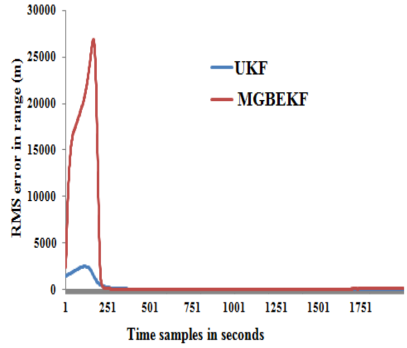

Figure 3: RMS Errors in Estimates of Range

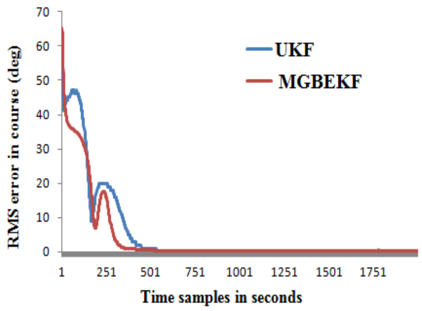

Figure 4: RMS Errors in Estimates of Course

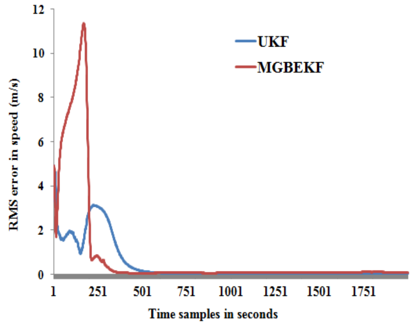

Figure 5: RMS Error in Estimates of Speed

4. Conclusion

An attempt is made to present the analysis of MGBEKF and UKF algorithms for bearings-only object tracking. This is a crucial area of future research. Numerous scenarios were tested using Monte-Carlo simulations. But only few scenarios have been presented here which are sufficient to indicate the capability of MGBEKF over UKF.

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