DEVELOPMENT OF DESIGN CHARTS OF A PARALLELOGRAM SHAPED SLAB BY USING YIELD LINE METHODOLOGY

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ABSTRACT

Reinforced concrete parallelogram shaped slabs are widely used in bridge construction when the roads cross the water streams and canals at angles other than 90 degrees. They are also used in floor systems of reinforced concrete buildings as well as load bearing buildings where the floors and roofs are parallel framed for architectural reasons or space limitations. In the present work, the load-deflection behavior of simply supported reinforced concrete parallelogram shaped slabs was analyzed using experimental and analytical methods. Finite element modeling of eighteen numbers of simply supported reinforced concrete parallelogram shaped slabs were made and non-linear analysis carried out on all parallelogram shaped slabs to obtain the load-deflection relationship under the action of uniformly distributed load. To carry out this analysis, a free MS Office (MS Excel sheet) was used. Also, this can be compared with an experimental load deflection curves and using theoretical method, with this results .

Key words: Analysis, concrete, deflections, simply supported, parallelogram shaped slabs, and parallelogram shaped plates, slabs.

1. INTRODUCTION:

parallelogram shaped slab can be defined as a four sided slab having equal opposite angles other than 90^0.Parallelofram angle is usually measured clock wise from the vertical line perpendicular to the support line of the parallelogram shaped slab. The analysis and design of parallelogram shaped slab is very complicated due to its shape. At present, the majority of reinforced slab designs are based on ultimate limit state principles. Johansen's Yield line analysis is permitted by many codes of practice to determine the Collapse load. British and Indian codes of practice have also recommended the coefficients for the design values for the restrained rectangular slab, based on yield line analysis. Number of researchers have given design table of moment coefficient, based on yield line theory, for design of rectangular slabs with and without opening.

A number of researchers[3,5,6] have analyzed parallelogram shaped slabs analytically and experimentally. They are either for axial loads or concentrated load. Also, the support conditions and the aspect ratio of the slabs. In this thesis reinforced concrete parallelogram shaped slabs with different boundary conditions(4 cases)have been analyzed using yield line theory,

2.1. Review of Literature:

In this work the virtual work equation is formulated by considering the total slab into four segments namely A,B,C & D and considering the horizontal propagation of yield lines as C_1, C_2 and vertical yield line propogation as C_3, C_4. The virtual work equation is formed for the individual work done by the segments and finally the total work done is equated to the individual energy absorbed by the yield lines of each segment are derived, finally total energy absorbed by the yield lines .For carrying out this work the following references are considered and studied to derive the work equation

Veerendra kumar[10] has analysed Reinforcedparallelogram shapedslabs with different boundary conditions. The ratio of the span moment to support moments are kept equal to 0.75. and those values are compared with the codal values(IS: 456-2000).

Veerendra kumar[10] has analysed Reinforced skew slabs with different boundary conditions. The ratio of the span moment to support moments are kept equal to 0.75. and those values are compared with the codal values(IS: 456-2000).

K.U.Muthu[5] has given a breif review on strength, deflection and cracking of skew reinforced slabs. He carried out his study considering the changes occured to the slab are studied in two membrane actions. They are Compressive membrane and Tensile membrane.

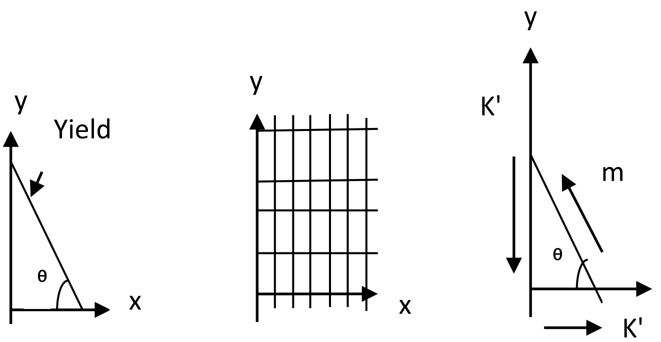
S.V.Dinesh[11] has analysed the effect of Skew angle on the single span reinforced bridge by using Finite Element method and also investigations are done only on the deck without edge beams. The FEM results are compared with the IRC loadings. A total of 90 bridge models are analysed.

A.kabir[7],Both experimental and numerical study has been carried out to investigate the effects of reinforcement arragements on the ultimate behaviour of skew slabs. A total of four skew slabs were expermentally tested in the laboratory. All the slabs were identical in dimension except the reinforcement arrangements.

K.Rambabu[9] has presented yield line analysis of two way rectangular slabs withopenings at different positions and supported on four sides with different boundary conditions. He has coverd the limitations of Zaslavsky and Islam. In slabs the support reinforcement may or may not be equal on opposite sides of the slab. It depends on the distribution of moments. But K.Rambabu[9] has taken the orthogonal moment co-efficient for support reinforcement on both sides of slab is same.

2.2. JOHANSEN'S YIELD CRITERION:

Johnson[1] yield criterion is convenient and provides a good model of yield line behavior. The same criterion is used in this thesis. The Johansen[1]'s yield criterion can be written as



(a)Yield line Vectors (b) Reinforcemen (c) Moment

$m^-_n = I_1(I_3)\ m\ Cos^2\theta + I_2(I_4)\ m\ Sin^2\theta$ $\sum K = K^1_x + I_1(I_3) + K^1_y + I_2(I_4)$

$m^+_n = K^1_y m\ Cos^2\theta + K^1_x\ mSin^2\theta$

Where

m = Ultimate Moment per unit length of slab

K^1_y = Orthogonal moment coefficient for positive reinforcement running Parallel to y-axis

K^1_x = Orthogonal moment coefficient for positive reinforcement running Parallel to x-axis

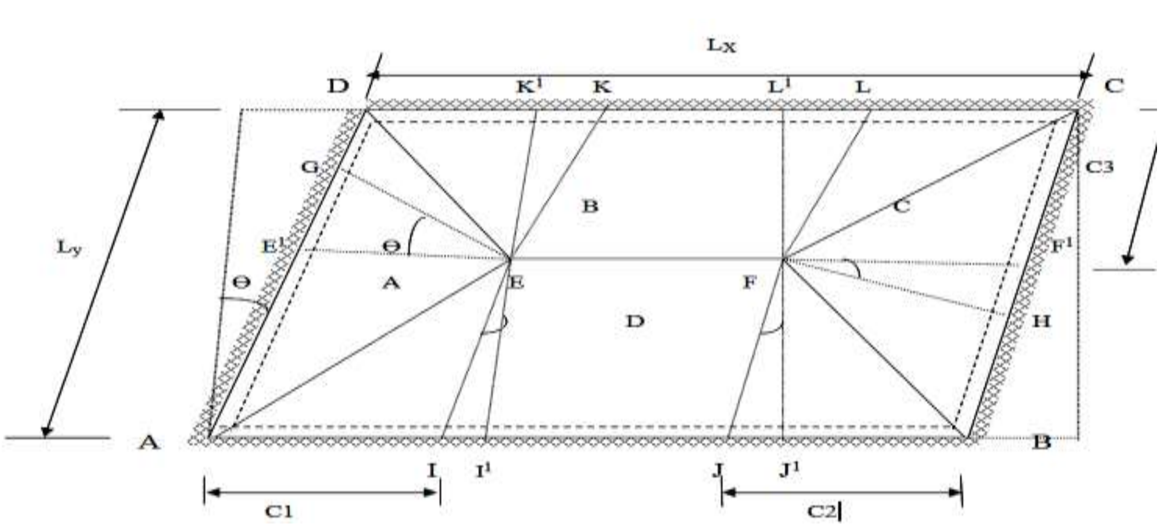
$I_2\ (I_4)$ = Orthogonal moment coefficient for negative reinforcement running Parallel to Y-axis on either side of the slab.

$I_1\ (I_3)$ = Orthogonal moment coefficient for negative reinforcement running Parallel to X-axis on either side of the slab.

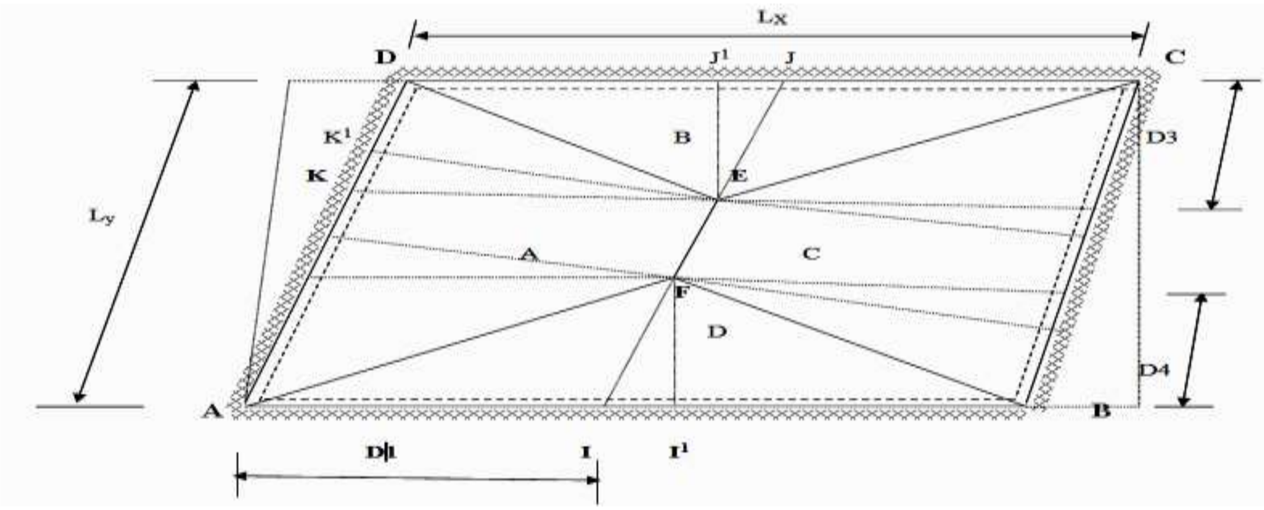
2.2. Virtual Work Equations for Continuous Slab (CS):

Consider the failure pattern and let δ be the small virtual displacement at. Three unknown dimensions D_1,D_2 and D_3 are necessary to define the yield line propagation completely.

As per virtual work equation



PATTERN-1



PATTERN-2

PATTERN-1

Work done by segment-A:

= (1/2)*AD*GE*(1/3)*W

$= \left[\frac{1}{2} \times L_y \times D_1 \times \cos\theta \times \frac{1}{3} \times W\right]$

$=\frac{1}{6} \times D_1 \times L_y \times \cos\theta \times W$

$=\frac{W}{6} \times C_1 \times L_Y \times \cos\theta$ … (1)

Work done by segment-B:

$=\left[\frac{1}{2} \times DK_1 \times K_1 \times \frac{1}{3} \times W\right] + \left[K_1E \times EF \times \frac{1}{2} \times W\right] + \left[\frac{1}{2} \times CL_1 \times FL_1 \times \frac{1}{3} \times W\right]$

$=\left[\frac{1}{2} \times (D_1 - D_3 \times \sin\theta) \times (D_3 \times \cos\theta \times \frac{W}{3}\right] + \left[(L_x - D_1 - D_2) \times (D_3 \times \cos\theta) \times \frac{W}{3}\right] + \left[\frac{1}{2} \times (D_2 + D_3 \times \sin\theta) \times (D_3 \times \cos\theta) \times \frac{W}{3}\right]$ … (2)

Work done by segment-C:

$= \left[\frac{1}{2} \times CB \times FH \times \frac{1}{3} \times W\right]$

$= \left[\frac{1}{2} \times L_y \times D_2 \times \cos\theta \times \frac{W}{3}\right]$

$= \left[\frac{W}{6} \times L_y \times \cos\theta \times D_2\right]$ … (3)

Work done by segment-D:

$$= \left[\frac{1}{2} \times AI_1 \times EI_1 \times \frac{1}{3} \times W\right] + \left[\frac{1}{2} \times I_1J_1 \times EI_1 \times W\right] + \left[\frac{1}{2} \times BJ_1 \times FJ_1 \times \frac{1}{2} \times W\right]$$

$$=\left[\frac{1}{2} \times \left(D_1 + \left(L_y - D_3\right) \times \sin\theta\right) \times \left(\left(L_y - D_3\right) \times \cos\theta\right) \times \frac{W}{3}\right] + \left[\left(L_x - D_1 - D_2\right) \times \left(L_y - D_3\right) \times \cos\theta \times \frac{W}{2}\right] + \left[\frac{1}{2} \times \left(D_2 - \left(L_y - D_3\right)\sin\theta\right) \times \left(\left(L_y - D_3\right)\cos\theta\right) \times \frac{W}{3}\right]$$

$$=\left[\left(L_y - D_3\right) \times \cos\theta \times \frac{W}{6} \times \left(D_1 + D_2\right)\right] + \left[\left(L_y - D_3\right) \times \cos\theta \times \frac{W}{2} \times \left(L_x - D_1 - D_2\right)\right] \qquad \dots (4)$$

TOTAL WORK DONE = SEGMENTS (A+B+C+D)

$$= \frac{W}{6} \times D_1 \times L_Y \times \cos\theta + \left[\frac{1}{2} \times (D_1 - D_3 \times \sin\theta) \times (D_3 \times \cos\theta \times \frac{W}{3}\right] + \left[(L_x - D_1 - D_2) \times (D_3 \times \cos\theta) \times \frac{W}{3}\right] + \left[\frac{1}{2} \times (D_2 + D_3 \times \sin\theta) \times (D_3 \times \cos\theta) \times \frac{W}{3}\right] + \left[\frac{W}{6} \times L_y \times \cos\theta \times D_2\right] + \left[\left(L_y - D_3\right) \times \cos\theta \times \frac{W}{6} \times (D_1 + D_2)\right] + \left[\left(L_y - D_3\right) \times \cos\theta \times \frac{W}{2} \times (L_x - D_1 - D_2)\right] \qquad \dots (5)$$

By solving the above equation

TOTAL WORK DONE $= wL^2{}_y \times r \times \cos\theta \left[\frac{1}{2} - \frac{1}{6r_1} - \frac{1}{6r_2}\right]$ … (6)

Energy Absorbed by Yield lines:

$$=\left[m(K_x{}^1 \cos^2\theta + K_y{}^1 \sin^2\theta) \times \frac{L_y}{C_2\cos\theta}\right] + \left[m(K_x{}^1 + K_y{}^1 \sin^2\theta) \times \frac{L_y}{D_1\cos\theta}\right] + \left[K_Y{}^1 \times m \times \frac{L_x}{D_3\cos\theta} + I_2 \times m \times \frac{L_x}{D_3\cos\theta} + m(K_x{}^1 \cos^2\theta + K_y{}^1 \sin\theta) \times \frac{L_Y}{D_2\cos\theta} + m(K_x{}^1 \cos^2\theta + K_y{}^1 \sin^2\theta) \times \frac{L_Y}{D_2\cos\theta} + K_Y{}^1 \times m \times \frac{L_x}{(L_y - D_3)\times\cos\theta} + I_4 \times m \times \frac{L_x}{(L_y - D_3)\times\cos\theta}\right]$$

$$=\frac{M}{\cos\theta}\begin{bmatrix} \frac{(k_x{}^1 + I_1) \times (r_1 \times \cos^2\theta)}{r} + \frac{(k_y{}^1 + I_2) \times (r_1 \times \sin^2\theta)}{r} + \\ ((k_y{}^1 + I_2) \times (r_3 \times r)) + \frac{(k_x{}^1 + I_3) \times (r_2 \times \cos^2\theta)}{r} + \\ \frac{(k_y{}^1 + I_4) \times (r_2 \times \sin^2\theta)}{r} + \frac{(k_y{}^1 + I_4) \times (r_3 \times r)}{(r_3 - 1)} \end{bmatrix}$$

… (7)

By principle of virtual work

$$\frac{wl_{y^2}}{m}=\frac{\left[\begin{array}{c}\frac{(k_x{}^1+I_1)\times(r_1\times\cos^2\theta)}{r}+\frac{(k_y{}^1+I_2)\times(r_1\times\sin^2\theta)}{r}+((k_y{}^1+I_2)\times(r_3\times r))+\\ \frac{(k_x{}^1+I_3)\times(r_2\times\cos^2\theta)}{r}+\\ \frac{(k_y{}^1+I_4)\times(r_2\times\sin^2\theta)}{r}+\frac{(k_y{}^1+I_4)\times(r_3\times r)}{(r_3-1)}\end{array}\right]}{wl^2{}_y\times r\times\cos\theta\left[\frac{1}{2}-\frac{1}{6r_1}-\frac{1}{6r_2}\right]} \quad \ldots (8)$$

2.2.1. Virtual Work Equations for SIMPLY SUPPORTED Slab (SS):

PATTERN-1

$$\frac{wl_{y^2}}{m}=\frac{\left[\begin{array}{c}\frac{(k_x{}^1)\times(r_1\times\cos^2\theta)}{r}+\frac{(k_y{}^1)\times(r_1\times\sin^2\theta)}{r}+((k_y{}^1)\times(r_3\times r))+\\ \frac{(k_x{}^1)\times(r_2\times\cos^2\theta)}{r}+\frac{(k_y{}^1)\times(r_2\times\sin^2\theta)}{r}+\frac{(k_y{}^1)\times(r_3\times r)}{(r_3-1)}\end{array}\right]}{wL^2{}_y\times r\times\cos\theta\left[\frac{1}{2}-\frac{1}{6r_1}-\frac{1}{6r_2}\right]} \quad \ldots (9)$$

PATTERN-2

$$\frac{wl_{y^2}}{m}=\left[\frac{\left[\frac{(K^1{}_x\cos^2\theta+K^1{}_y\sin^2\theta)r^2}{r\times(r_1-1)}+\left[\frac{\left[K_y{}^1\times r\right]\times[r_3+r_4]}{\cos\theta}\right]+r_4\right]}{r\times\cos^2\theta\times(\frac{1}{2}-\frac{1}{6r_3}-\frac{1}{6r_4})}\right] \quad \ldots (10)$$

2.2.2. Virtual Work Equations for TWO LONG EDGE CONTINIOUS Slab (TLCS):

PATTERN-1

$$\frac{wl_{y^2}}{m}=\frac{\left[\begin{array}{c}\frac{(k_x{}^1)\times(r_1\times\cos^2\theta)}{r}+\frac{(k_y{}^1+I_2)\times(r_1\times\sin^2\theta)}{r}+((k_y{}^1+I_2)\times(r_3\times r))+\\ \frac{(k_x{}^1)\times(r_2\times\cos^2\theta)}{r}+\frac{(k_y{}^1+I_4)\times(r_2\times\sin^2\theta)}{r}+\frac{(k_y{}^1+I_4)\times(r_3\times r)}{(r_3-1)}\end{array}\right]}{wL^2{}_y\times r\times\cos\theta\left[\frac{1}{2}-\frac{1}{6r_1}-\frac{1}{6r_2}\right]}$$

… (11)

PATTERN-2

$$\frac{wl_{y^2}}{m}=\left[\frac{\left[\begin{array}{c}\frac{(K^1{}_x\cos^2\theta+K^1{}_y\sin^2\theta)r^2}{r\times(r_1-1)}+\left[\frac{\left[K_y{}^1\times r\right]\times[r_3+r_4]}{\cos\theta}\right]+\\ r_1\left[\frac{I_2\sin^2\theta}{r}+\frac{I_4\sin^2\theta}{(r_1-1)\times r}\right]+r\left[(I_2\times r_3)+(I_4+r_4)\right]\end{array}\right]}{r\times\cos^2\theta\times(\frac{1}{2}-\frac{1}{6r_3}-\frac{1}{6r_4})}\right] \quad \ldots (12)$$

2.2.3. Virtual Work Equations for TWO SHORT EDGE CONTINIOUS Slab (TSCS):

PATTERN-1

$$\frac{wl_{y^2}}{m}=\frac{\left[\begin{array}{c}\frac{(k_x{}^1+I_1)\times(r_1\times\cos^2\theta)}{r}+\frac{(k_y{}^1)\times(r_1\times\sin^2\theta)}{r}+((k_y{}^1)\times(r_3\times r))+\\ \frac{(k_x{}^1+I_3)\times(r_2\times\cos^2\theta)}{r}+\frac{(k_y{}^1)\times(r_2\times\sin^2\theta)}{r}+\frac{(k_y{}^1)\times(r_3\times r)}{(r_3-1)}\end{array}\right]}{wL^2{}_y\times r\times\cos\theta\left[\frac{1}{2}-\frac{1}{6r_1}-\frac{1}{6r_2}\right]}$$

… (13)

PATTERN-2

$$\frac{wl_{y^2}}{m}=\left[\frac{\left[\frac{(K^1{}_x\cos^2\theta+K^1{}_y\sin^2\theta)r^2}{r\times(r_1-1)}+\left[\frac{\left[K_y{}^1\times r\right]\times[r_3+r_4]}{\cos\theta}\right]+r_1\left[\frac{I_1\cos^2\theta}{r}+\frac{I_3\cos^2\theta}{(r_1-1)\times r}\right]\right]}{r\times\cos^2\theta\times\left(\frac{1}{2}-\frac{1}{6r_3}-\frac{1}{6r_4}\right)}\right] \quad \text{… (14)}$$

3.0. Design Charts

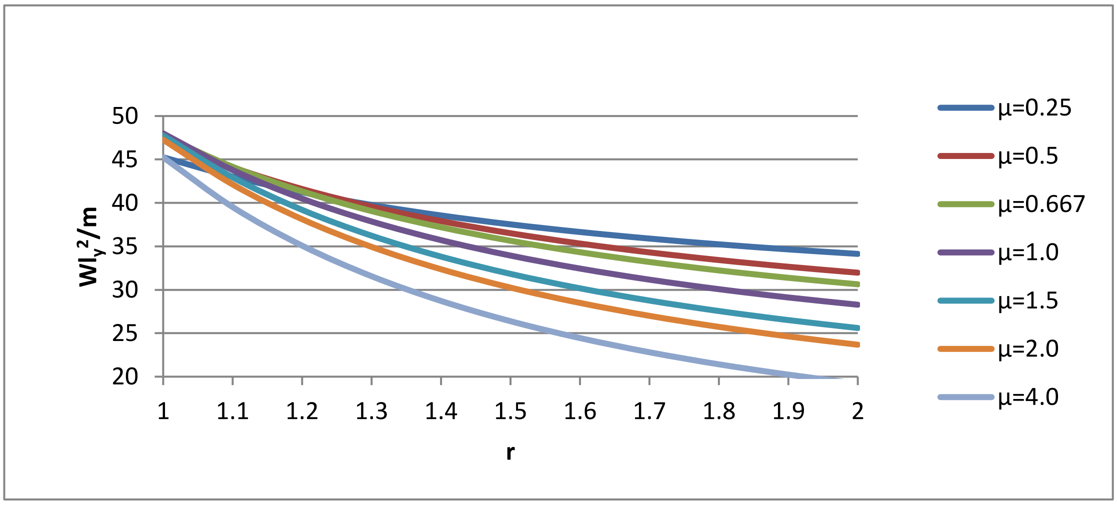


Chart-1: For various μwith respect to aspect ratio (r) & Θ=0

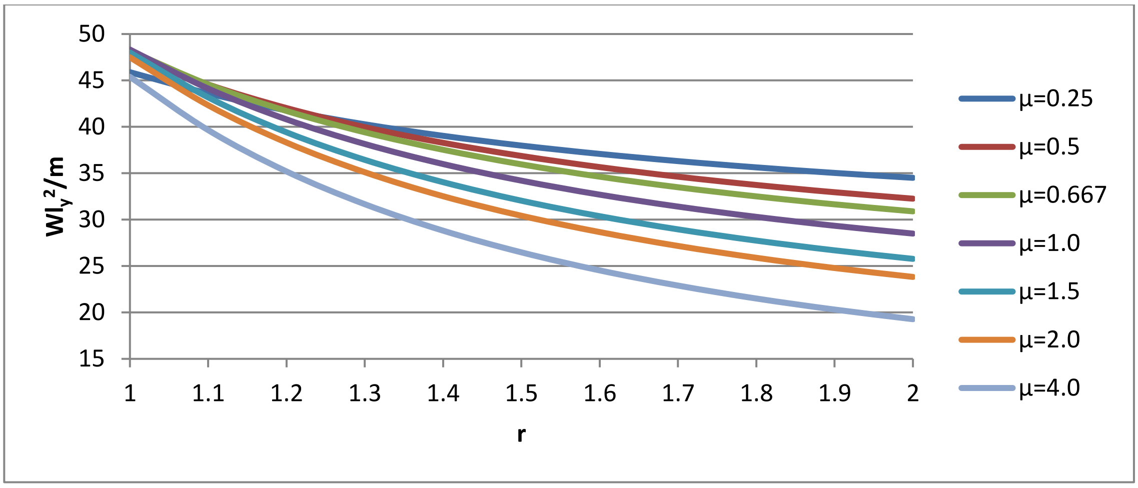


Chart-2: For various µwith respect to aspect ratio (r) & Θ=5

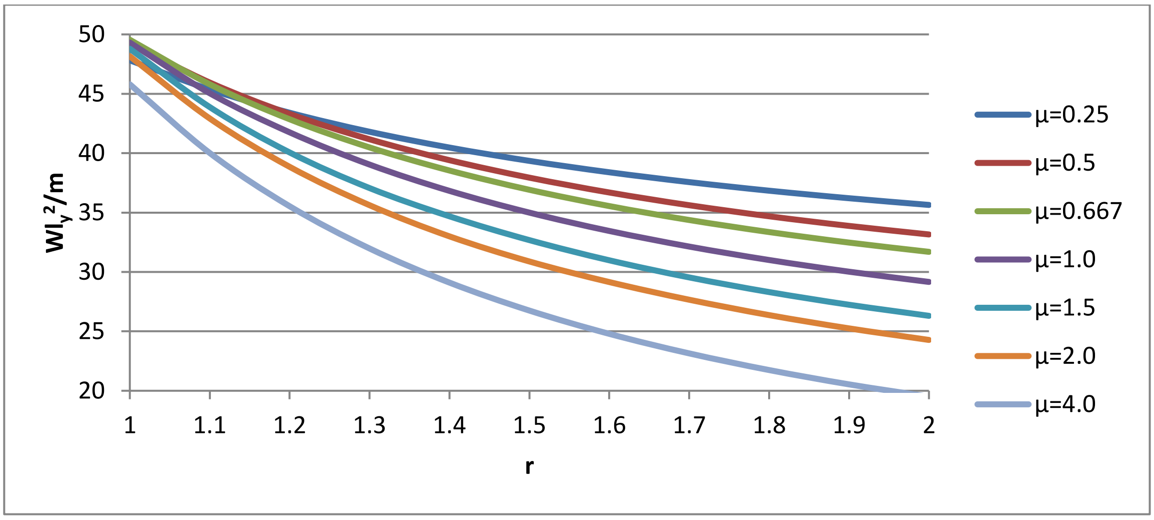


Chart-3: For various µwith respect to aspect ratio (r) & Θ=10

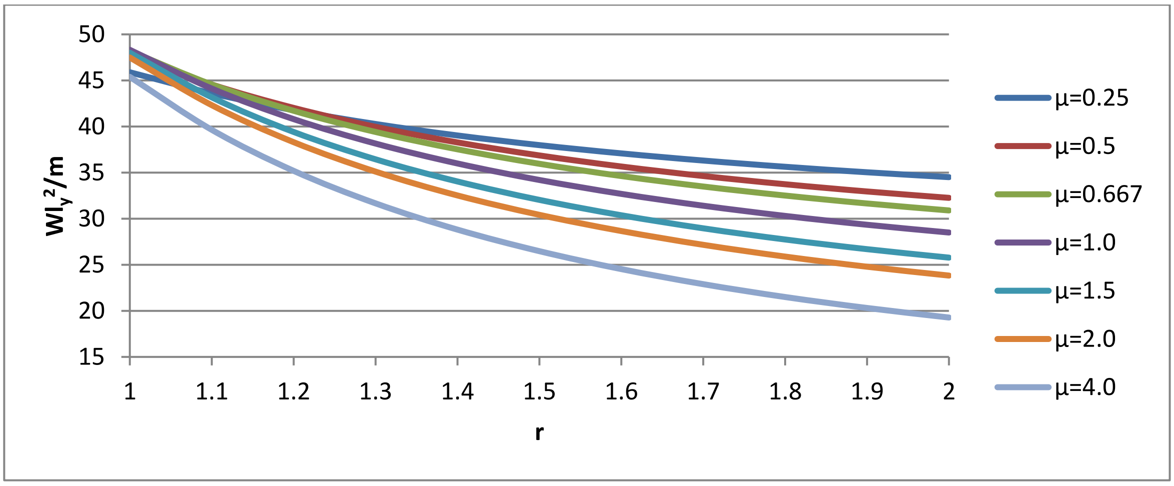


Chart-4: For various µwith respect to aspect ratio (r) & Θ=15

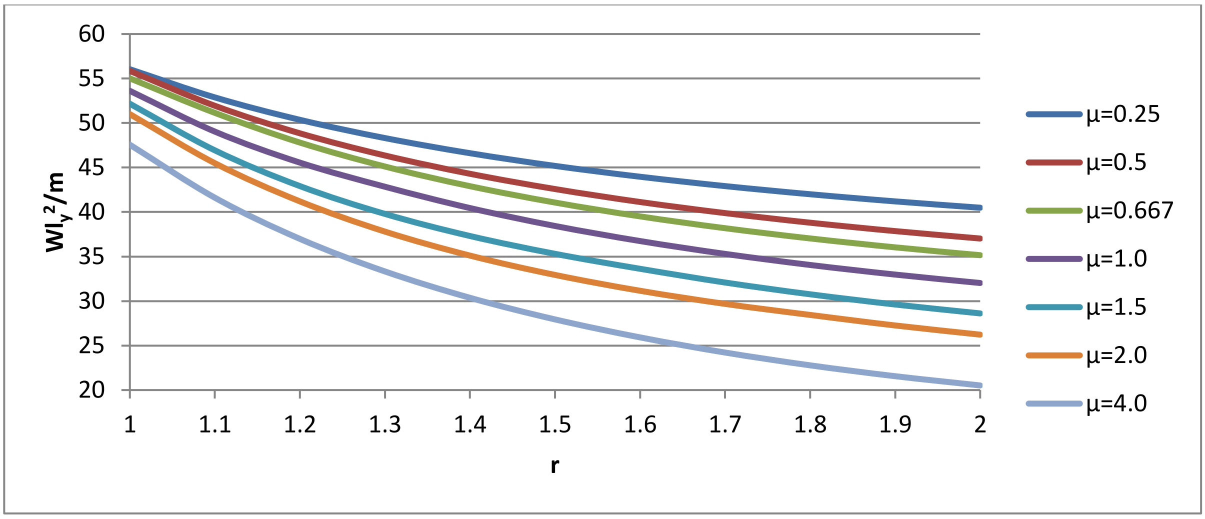


Chart-5: For various µwith respect to aspect ratio (r) & Ɵ=20

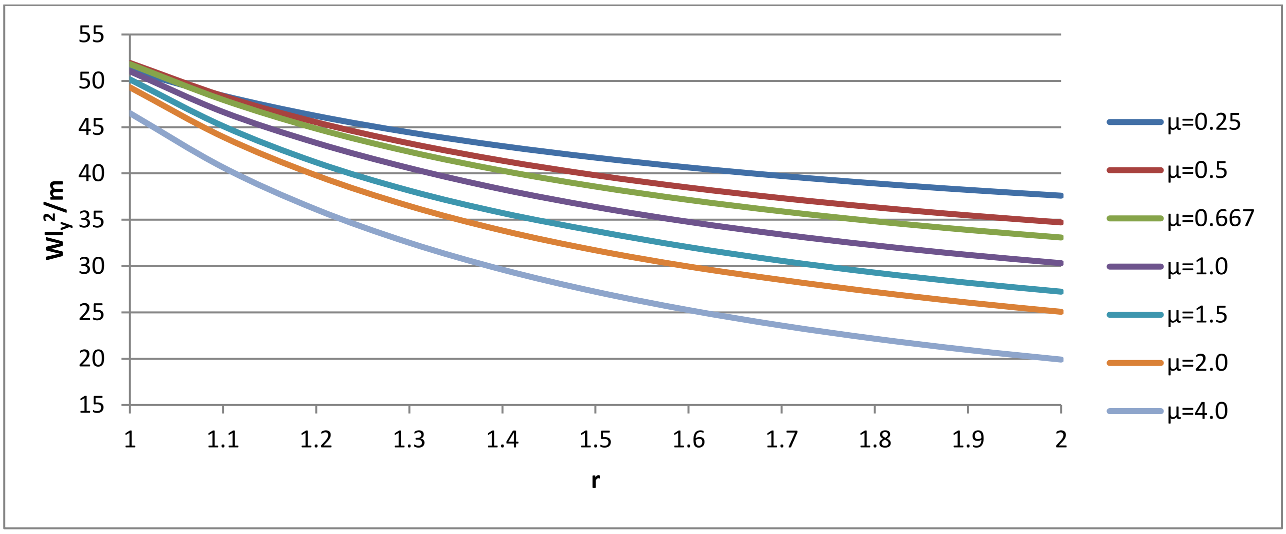


Chart-6: For various µwith respect to aspect ratio (r) & Ɵ=35

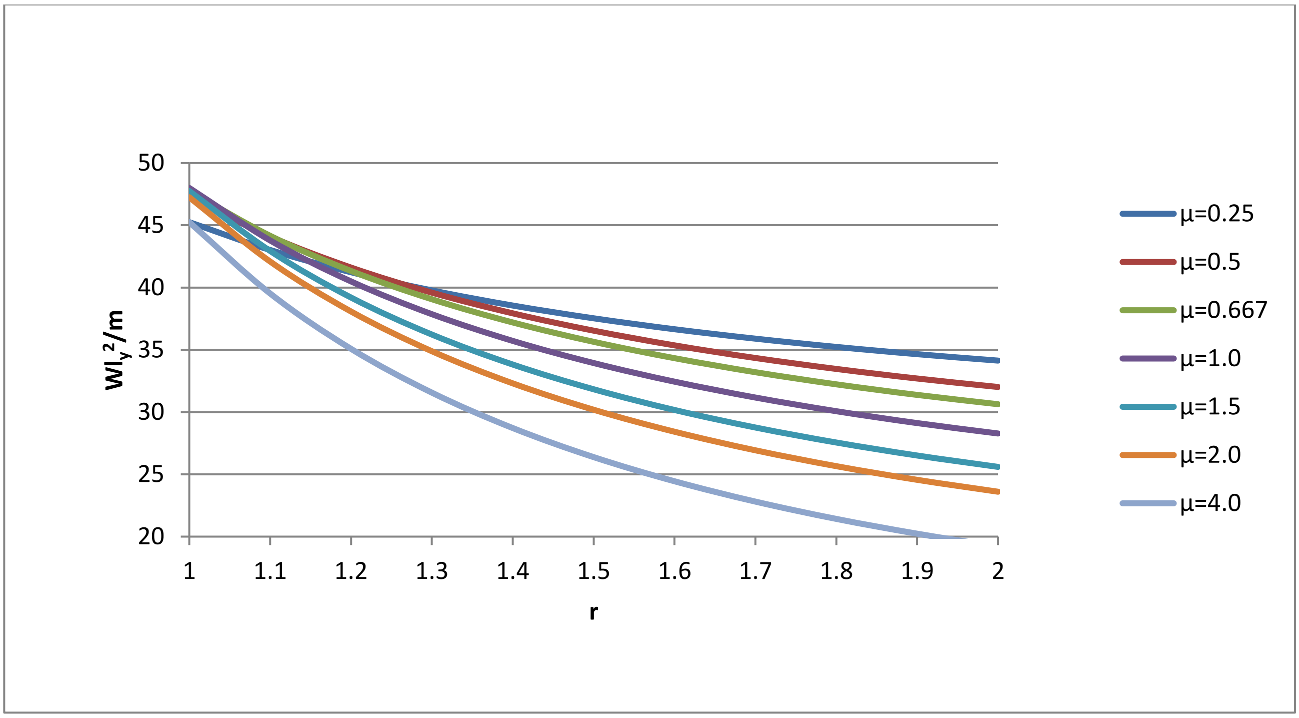


Chart-7: For various μwith respect to aspect ratio (r) & Θ=15

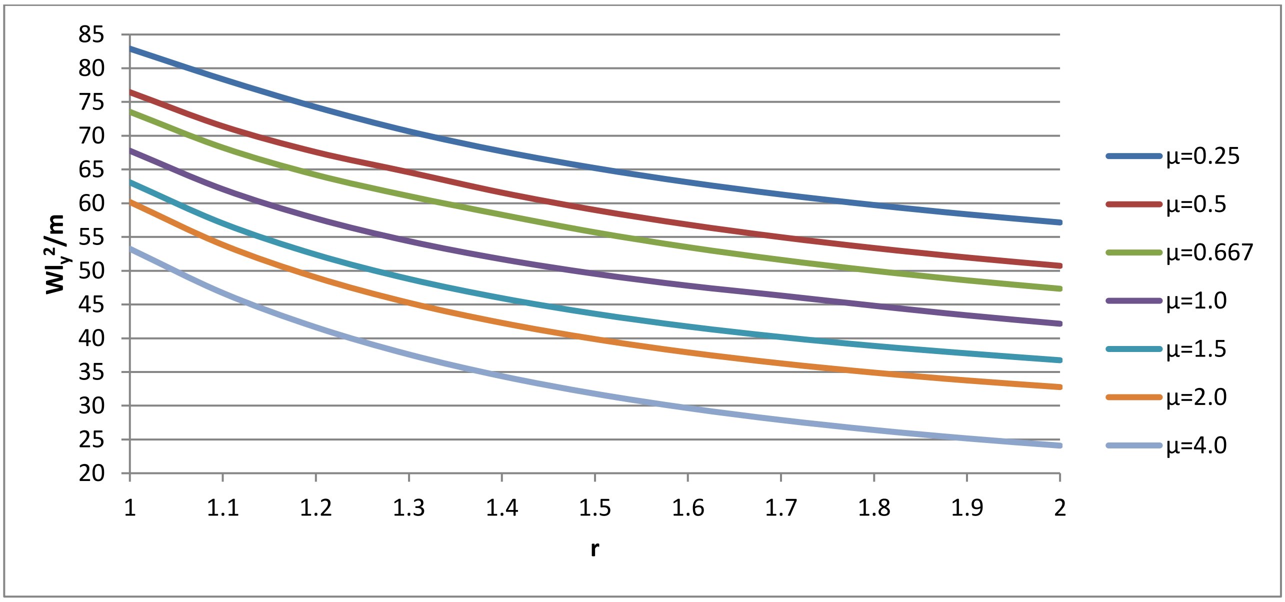


Chart-8: For various μwith respect to aspect ratio (r) & Θ=35

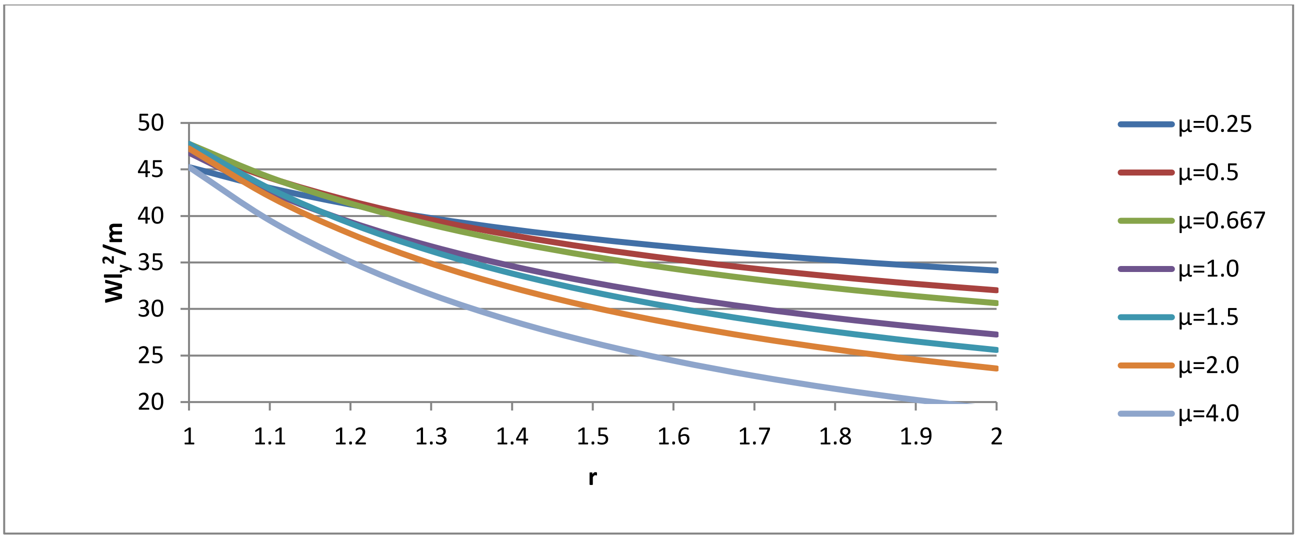


Chart-9: For various µwith respect to aspect ratio (r) & Θ=0

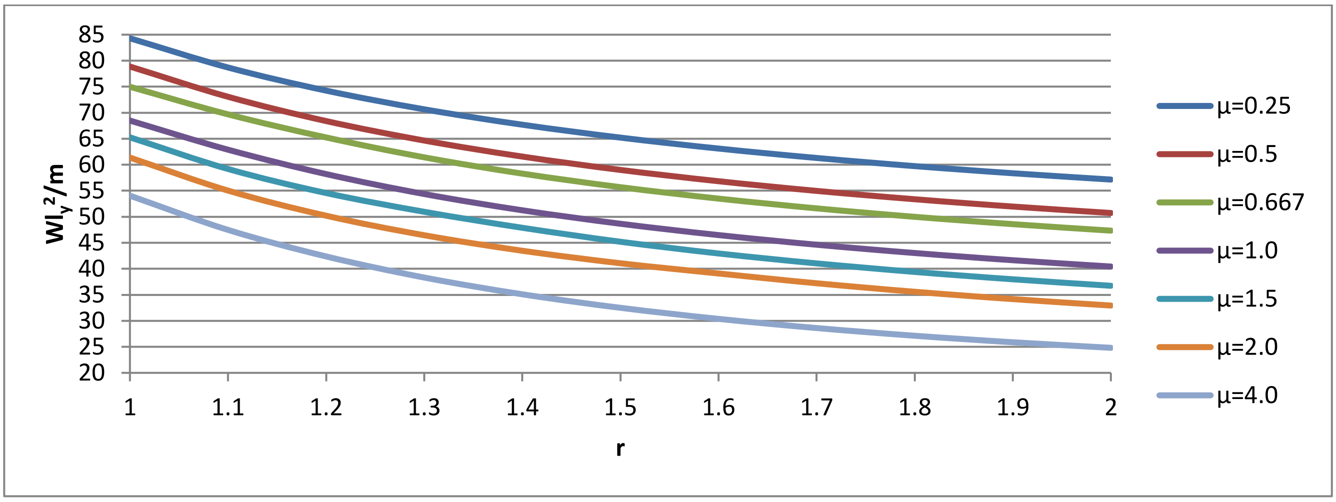


Chart-10: For various µwith respect to aspect ratio (r) & Θ=35

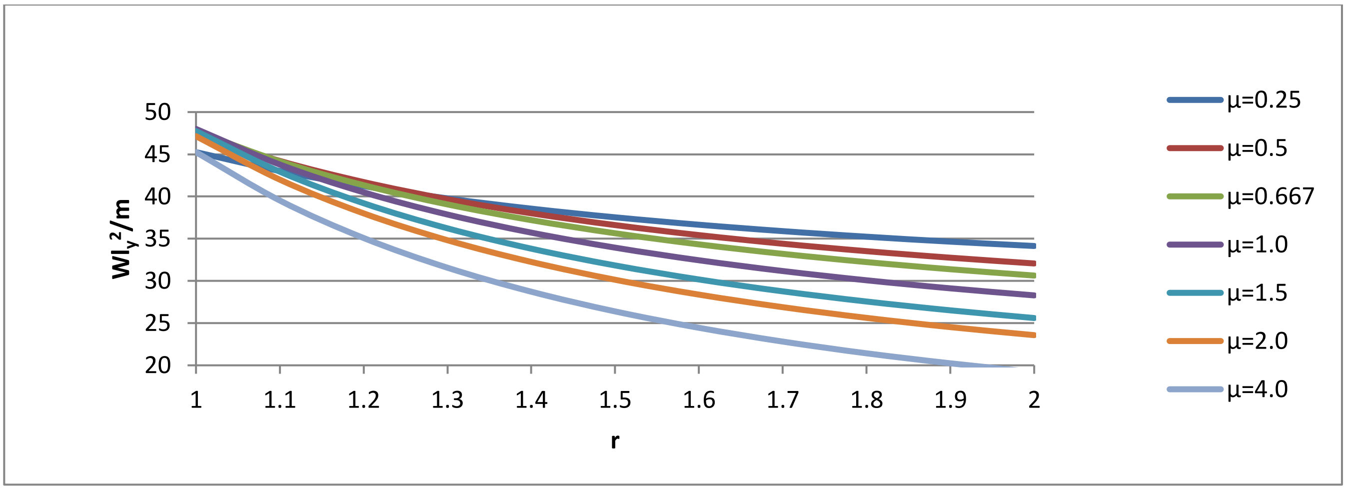


Chart-11: For various µwith respect to aspect ratio (r) & Θ=0

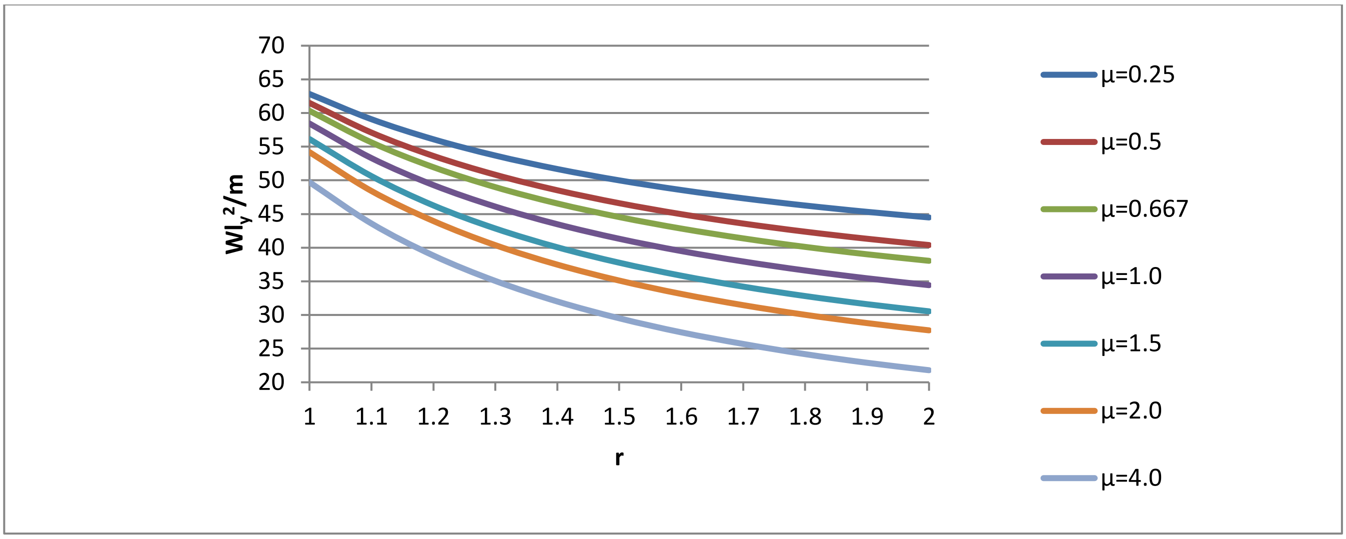


Chart-12: For various μwith respect to aspect ratio (r) & Ɵ=25

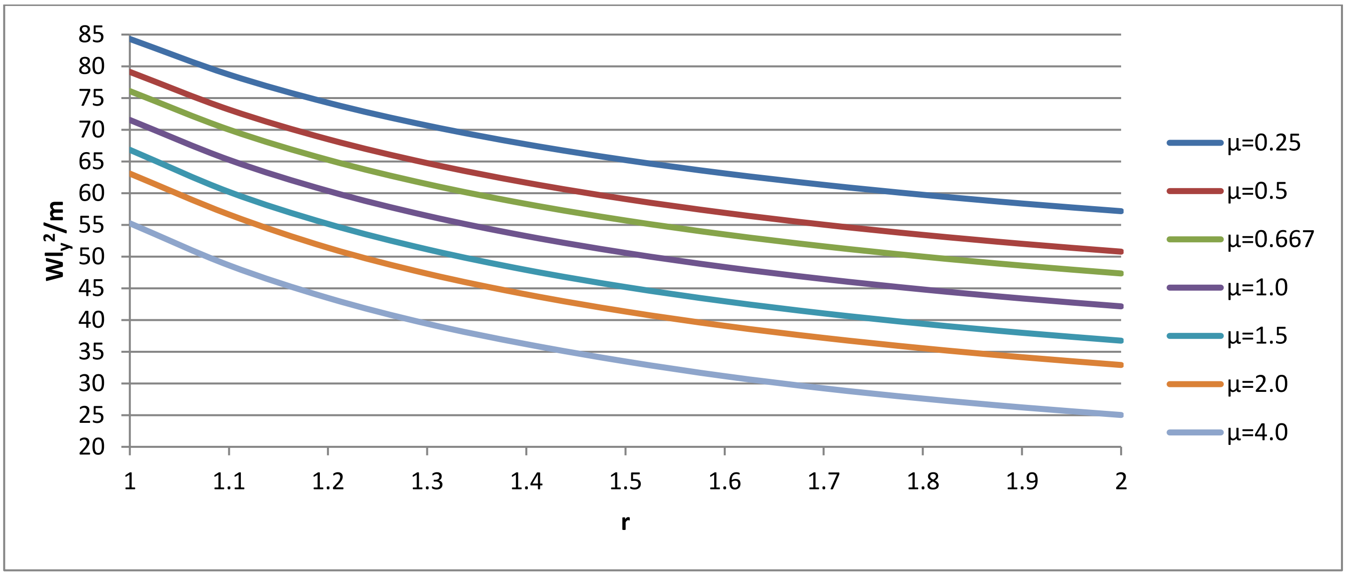


Chart-13: For various μwith respect to aspect ratio (r) & Ɵ=35

Conclusions

1. The two failure patterns are considered to assess the strength of the skew slab. With the four edge conditions namely- All sides continuous, two Short sides continuous, two long sides continuous, all sides simply supported.
2. A computer program is complied to analyze the two patterns for all the four edge conditions.
3. The strength variation criteria are observed through the graphs obtained.
4. The similarities between the coefficients are observed with respect to aspect ratio, skew angles for their strength values.
5. Scrutinized the same edge conditions with various coefficient values and also various angles 'Ɵ' with respect to their aspect ratio.
6. The charts illustrate the variations that can be noticed easily.
7. Good numbers of charts are presented to obtain a clear view to avoid ambiguity.
8. From chart -6 i.e. when the skew angle is 35^0 the strength values are same for μ=0.25, μ=0.5 & μ=0.667, when the aspect ratio ranges from 1.0 to 1.3.

9. If the skew angle is 15^0 it is noticed that the strength is same up to aspect ratio 1.4 which can be seen in the chart-7.
10. If we consider the Reverse designers coefficient then the strength values will be sane as that of Affine coefficients from charts 9.
11. For two short side continuous condition , the results are similar in the case of Affine coefficients which is illustrated from the chart-16 to 21.
12. When the skew angle is 35^0 the strength variation can be observed very high and it can be deduced from the charts-23.

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