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FS-Subsets Under The FS-Complement Operator

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Abstract: In this paper we search the nature of an image of an Fs-subset under an Fs-function whenever this Fs-function acts on complement of an Fs-subset .Also we prove that the image of the complemented Fs-subse tcontains complement of the image under some condition.

For any Fs-subsets $\mathcal{U}$, $\mathcal{V}$ and $\mathcal{P}$ of $\mathcal{W}$, with $\mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$ $\mathcal{V} = (V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V)$, $\mathcal{P} = (P_1, P, \overline{P}(\mu_{1P_1}, \mu_{2P}), L_P)$ and $\mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$

i) $\mathcal{U} \Delta \mathcal{V} = \mathcal{V} \Delta \mathcal{U}$

ii) $\mathcal{U} \Delta \Phi_{\mathcal{W}} = \mathcal{U}$

iii) $\mathcal{U} \Delta \mathcal{U} = =\Phi_{\mathcal{W}}$

iv) $\mathcal{U} \Delta (\mathcal{V} \Delta \mathcal{P}) = (\mathcal{U} \Delta \mathcal{V}) \Delta \mathcal{P}$ with condition provided $\mu_{1P_1}x = \mu_{2U}x = \mu_{1V_1}x$, $\mu_{1U_1} = \overline{V}x = \mu_{2P}x$

v) $(\mathcal{U} - \mathcal{V}) - \mathcal{P} = \mathcal{U} - (\mathcal{V} \sqcup \mathcal{P})$ with condition provided $(\overline{V}x \curlyvee \overline{P}x) \geq (\mu_{1P_1}x \curlywedge (\mu_{2V})^c)x \curlywedge (\mu_{1V_1}x \curlywedge (\mu_{2P}x)^c)$

Here $\mathcal{U} \Delta \mathcal{V}$ Stands for $(\mathcal{U} \sqcap \mathcal{V}^{C_{\mathcal{A}}}) \sqcup (\mathcal{U}^{C_{\mathcal{A}}} \sqcap \mathcal{V})$,where $\mathcal{U}^{C_{\mathcal{A}}}$ is the Fs-Complement of $\mathcal{U}$ in $\mathcal{W}$ and $\mathcal{U} - \mathcal{V}$ stands for $\mathcal{U} \sqcap \mathcal{V}^{C_{\mathcal{A}}}$

Here $\Phi_{\mathcal{W}}$ stands for Fs-empty set

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We include the necessary principles useful to read in this chapter . For more details of the results or statements in this chapter one can refer [Appendix-B]

Fs-set

1.1 Definition: Let $W \sqsubseteq W_1 \sqsubseteq X$ where X is a non-void universal set. Then a four tuple of the form $\mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$ is an Fs-set if and only if,

(1) L_W is a complete Boolean Algebra

(2) $\mu_{1W_1} : W_1 \to L_W$, $\mu_{2W} : W \to L_W$ are mappings such that $\mu_{1W_1}|W \geq \mu_{2W}$ i.e $\mu_{1W_1}x \geq \mu_{2W}x$ for each $x \in W$

(3) $\overline{W}$: $W \to L_W$ is defined by

$\overline{W}x = \mu_{1W_1}x \wedge (\mu_{2W}x)^c$ for each $x \in W$

Fs-subset

1.2 Definition: Suppose $\mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$ and $\mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$ are two Fs-sets. We say $\mathcal{U}$ is an Fs-subset of $\mathcal{W}$, in symbol, we write $\mathcal{U} \sqsubseteq \mathcal{W}$, if and only if

$U_1 \sqsubseteq W_1$, $W \sqsubseteq U$

A result related to containment - in fact, a characterization lemma

1.3 Lemma: If $\mathcal{U}$, $\mathcal{V}$ and $\mathcal{P}$ are Fs-subsets with $\mathcal{U} \sqsubseteq \mathcal{V}$ and $\mathcal{V} \sqsubseteq \mathcal{P}$, then $\mathcal{U} \sqsubseteq \mathcal{P}$

1.4 Remark: For some $L_\Omega, L_\Omega \leq L_W$, the specific object $\Omega_\varphi = (\Omega_1, \Omega, \overline{\Omega}(\mu_{1\Omega_1}, \mu_{2\Omega}), L_\Omega)$ with conditions

(a)$\Omega \not\sqsubseteq \Omega_1$ or Ω is a void set

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(b) $\mu_{1\Omega_1}x \not\geq \mu_{2\Omega}x$, for some $x \in \Omega \sqcap \Omega_1$ or $\mu_{2\Omega}$ is a void function i.e $\mu_{2\Omega}$ is a funcion with domain void set is called a Type-I void set and is denoted by φ_1 and throughout this thesis, this specific Ω_φ is denoted by φ_1 and we agree that $\varphi_1 \sqsubseteq \mathcal{U}$, for any Fs − subset U

1.5 Definition: If $\mathcal{Y} = (Y_1, Y, \overline{Y}(\mu_{1Y_1}, \mu_{2Y}), L_Y)$ is an Fs-subset of $\mathcal{U}$, with the following properties

(a') $\mathcal{U} \sqsubseteq \mathcal{W}$

(b') $Y_1 = Y = W$

(c') $L_Y \leq L_W$

(d') $\overline{Y} = 0$ or $\mu_{1Y_1} = \mu_{2Y}$

then, we say that $\mathcal{Y}$ is a Type-II Void set and is denoted by φ_2

1.6 Definitions of different kinds of equality of Fs-subsets:

Suppose $\mathcal{U}_1 = (U_{11}, U_1, \overline{U}_1(\mu_{1U_{11}}, \mu_{2U_1}), L_{U_1})$ and $\mathcal{U}_2 = (U_{12}, U_2, \overline{U}_2(\mu_{1U_{12}}, \mu_{2U_2}), L_{U_2})$ are two Fs-subsets. We say that $\mathcal{U}_1$ and $\mathcal{U}_2$ in (1), (2), (3),(4), (5) and (6) respectively

(1) Equality of 1st kind if $U_{11} = U_{12}$, $L_{U_1} = L_{U_2}$

(2) Equality of 2nd kind if $U_1 = U_2$, $L_{U_1} = L_{U_2}$

(3) Equality of 3rd kind if $\mathcal{U}_1$ and $\mathcal{U}_2$ are of equality of 1st kind with $\mu_{1U_{11}} = \mu_{1U_{12}}$

(4) Equality of 4th kind, if $\mathcal{U}_1$ and $\mathcal{U}_2$ are of equality of 2nd kind with $\mu_{2U_1} = \mu_{2U_2}$

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(5) Equality of total with the notation $\mathcal{U}_1 = \mathcal{U}_2(T)$, if $\mathcal{U}_1$ and $\mathcal{U}_2$ are of equality of 2nd kind with $\overline{U}_1 = \overline{U}_2$

(6) Full-equal, denoted $\mathcal{U}_1 = \mathcal{U}_2$, if $\mathcal{U}_1$ and $\mathcal{U}_2$ are of equality of 3rd kind and equality of 4th kind

The Operations Fs-union(⊔) and Fs-intersection (⊓)

1.7 Definition: Let $\mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$, $\mathcal{V} = (V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V) \sqsubseteq \mathcal{W}$. Then,

$\mathcal{U} \sqcup \mathcal{V} = \mathcal{P} = (P_1, P, \overline{P}(\mu_{1P_1}, \mu_{2P}), L_P)$, where

(1) $P_1 = U_1 \sqcup V_1$ (crispset union)

$P = U \sqcap V$ (crisp set intersection)

(2) $L_P = L_U \curlyvee L_V$ = The complete subalgebra generated by $L_U \sqcup L_V$

(3) $\mu_{1P_1} : P_1 \to L_P$ is defined by

$$\mu_{1P_1}x = (\mu_{1U_1} \curlyvee \mu_{1V_1})x = \begin{cases} \mu_{1U_1}x, if\ x \in U_1, x \notin V_1 \\ \mu_{1V_1}x, if\ x \in V_1, x \notin U_1 \\ \mu_{1U_1}x \curlyvee \mu_{1V_1}x, if\ x \in U_1, x \in V_1, \end{cases}$$

$\mu_{2P} : P \to L_P$ is defined by

$\mu_{2P}x = \mu_{2U}x \curlywedge \mu_{2V}x$ and

$\overline{P}: P \to L_P$ is defined by

$$\overline{P}x = \mu_{1P_1}x \curlywedge (\mu_{2P}x)^c$$

1.8 Definition: Let $\mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$ and $\mathcal{V} = (V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V) \sqsubseteq \mathcal{W}$ with the properties:

(i) $U_1 \sqcap V_1 \sqsupseteq U \sqcup V$

(ii) $\mu_{1U_1}x \curlywedge \mu_{1V_1}x \geq (\mu_{2U} \curlyvee \mu_{2V})x$ for each $x \in W$ (see the definition1.7)

Then,

$\mathcal{U} \sqcap \mathcal{V} = \mathcal{Q} = (Q_1, Q, \overline{Q}(\mu_{1Q_1}, \mu_{2Q}), L_Q)$, where

(1) $Q_1 = U_1 \sqcap V_1$, $Q = U \sqcup V$

(2) $L_Q = L_U \curlywedge L_V = L_U \sqcap L_V$

(3) $\mu_{1Q_1}: Q_1 \rightarrow L_Q$ is defined by

$$\mu_{1Q_1}x = \mu_{1U_1}x \curlywedge \mu_{1V_1}x$$

$\mu_{2Q} : Q \rightarrow L_Q$ is defined by

$$\mu_{2Q}x = (\mu_{2U} \curlyvee \mu_{2V})x$$

$\overline{Q}: Q \rightarrow L_Q$ is defined by

$\overline{Q}x = \mu_{1Q_1}x \curlywedge (\mu_{2Q}x)^c$.

Associative Laws

1.9 Proposition:

For $\mathcal{U}$, $\mathcal{V}$ and $\mathcal{P} \sqsubseteq \mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$, the following associative laws are true.

(I) $\mathcal{U} \sqcup (\mathcal{V} \sqcup \mathcal{P}) = (\mathcal{U} \sqcup \mathcal{V}) \sqcup \mathcal{P}$(full-equal)

(II) $\mathcal{U} \sqcap (\mathcal{V} \sqcap \mathcal{P}) = (\mathcal{U} \sqcap \mathcal{V}) \sqcap \mathcal{P}$ (full-equal)

1.10 Proposition: Suppose $\mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$, $\mathcal{V}=(V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V)$ and $\mathcal{P}=(P_1, P, \overline{P}(\mu_{1P_1}, \mu_{2P}), L_P)$.Then $\mathcal{U} \sqcup (\mathcal{V} \sqcap \mathcal{P})$ and $(\mathcal{U} \sqcup \mathcal{V}) \sqcap (\mathcal{U} \sqcup \mathcal{P})$ are full-equal, provided $\mathcal{V} \sqcap \mathcal{P}$ exists or $\mathcal{V} \sqcap \mathcal{P} \neq \varphi_1$

1.11 Remark: Whenever $\mathcal{V} \sqcap \mathcal{P} = \varphi_1$ this Law fails.

1.12 Proposition: Let $\mathcal{U} =(U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$, $\mathcal{V}=(V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V)$ and $\mathcal{P}=(P_1, P, \overline{P}(\mu_{1P_1}, \mu_{2P}), L_P)$. Then $\mathcal{U} \sqcap (\mathcal{V} \sqcup \mathcal{P})$and $(\mathcal{U} \sqcap \mathcal{V}) \sqcup (\mathcal{U} \sqcap \mathcal{P})$ full-equal provided

$$(\mathcal{U} \sqcap \mathcal{V}) \sqcup (\mathcal{U} \sqcap \mathcal{P}) \neq \varphi_1$$

Whenever $(\mathcal{U} \sqcap \mathcal{V}) \sqcup (\mathcal{U} \sqcap \mathcal{P}) = \varphi_1$ this law fails.

Fs-complement of an Fs-subset
1.13 Definition
Consider a particular Fs-set$\mathcal{W} = (W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$,$W \neq \Phi$,where

(i) $W \sqsubseteq W_1$

(ii) $L_W = [0, M_A]$ the complete Boolean algebra, M_A is the largest element of L_W

(iii) $\mu_{1W_1} = M_A, \mu_{2W} = 0$

$$\overline{W}x = \mu_{1W_1}x \wedge (\mu_{2W}x)^c = M_A \text{ for each } x \in W$$

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Given $\mathcal{U}$=$(U_1, U, \overline{U}(\mu_{1U_1},\mu_{2U}), L_U)$. We define Fs-complement of $\mathcal{U}$ in $\mathcal{W}$, denoted by $\mathcal{U}^{C_\mathcal{A}}$ for U=W and $L_U = L_W$ as

$\mathcal{U}^{C_\mathcal{A}} = \mathcal{P}$=$(P_1, P, \overline{P}(\mu_{1P_1}, \mu_{2P}), L_P)$, where

(a') $P_1 = C_A U_1 = U_1^c \sqcup W, P = U = W$ where $U_1^c = W_1 - U_1$

(b') $L_P = L_W$

(c') $\mu_{1P_1}: P_1 \to L_W$ is defined by

$$\mu_{1P_1}x = M_A$$

$\mu_{2P}: W \to L_W$ is defined by

$$\mu_{2P}x = \overline{U}x = \mu_{1U_1}x \wedge (\mu_{2U}x)^c$$

$\overline{P}: W \to L_W$ is defined by

$$\overline{P}x = \mu_{1P_1}x \wedge (\mu_{2P}x)^c = M_A \wedge (Ux)^c = (\overline{U}x)^c.$$

Fs-DeMorgan Laws of a pair of Fs-subsets

1.14 Proposition

For any pair of Fs-sets $\mathcal{U}$=$(U_1, U, \overline{U}(\mu_{1U_1},\mu_{2U}), L_U)$ and $\mathcal{V} = (V_1, V, \overline{V}(\mu_{1V_1}, \mu_{2V}), L_V)$ with $U = V = W$ and $L_U = L_V = L_W$, we can have

(1)$(\mathcal{U} \sqcup \mathcal{V})^{C_\mathcal{A}} = \mathcal{U}^{C_\mathcal{A}} \sqcap \mathcal{V}^{C_\mathcal{A}}$ if

$$(\overline{U}x)^c \wedge (\overline{V}x)^c \leq [(\mu_{1U_1}x)^c \vee \mu_{2V}x] \wedge [(\mu_{1V_1}x)^c \vee \mu_{2U}x] \text{for each } x \in W$$

(2)$(\mathcal{U} \sqcap \mathcal{V})^{C_\mathcal{A}} = \mathcal{U}^{C_\mathcal{A}} \sqcup \mathcal{V}^{C_\mathcal{A}}$, whenever $\mathcal{U} \sqcap \mathcal{V} \neq \Phi_\mathcal{W}$ =Fs-empty set of first kind

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1.15 Proposition: A family $\mathfrak{G}$ of all Fs-subsets $\mathcal{U} \sqsubseteq \mathcal{W}$ and $\mathcal{W}$ = $(W_1, W, \overline{W}(\mu_{1W_1}, \mu_{2W}), L_W)$

with $\mathcal{U} = (U_1, U, U(\mu_{1U_1}, \mu_{2U}), L_U)$ with U=W and $L_U = L_W$ is conditionally a commutative group with the operation Δ as defined below $\mathcal{U} \Delta \mathcal{V} = (\mathcal{U} \sqcap \mathcal{V}^{C_\mathcal{A}}) \sqcup (\mathcal{U}^{C_\mathcal{A}} \sqcap \mathcal{V})$ where $\mathcal{U}$ & V are Fs- subsets of $\mathcal{W}$, this proposition follows from 1.16 to 1.17.

1.16 Proposition: We can easily observe that for any Fs-subsets $\mathcal{U}$ & V, the following results are true

1) $\mathcal{U} \Delta \mathcal{V} = \mathcal{V} \Delta \mathcal{U}$

2) $\mathcal{U} \Delta \Phi_{\mathcal{W}} = \mathcal{U}$

3) $\mathcal{U} \Delta \mathcal{U} = \ = \Phi_{\mathcal{W}}$

1.17 Proposition :For any Fs-subsets $\mathcal{U}, \mathcal{V}$ and $\mathcal{P}$ of $\mathcal{W}$ the following

$\mathcal{U} \Delta (\mathcal{V} \Delta \mathcal{P}) = (\mathcal{U} \Delta \mathcal{V}) \Delta \mathcal{P}$ is true provided

a) $\mu_{1P_1}x = \mu_{2U}x = \mu_{1V_1}x$,

b) $\mu_{1U_1} = \overline{V}x = \mu_{2P}x$

Proof:

We have $\mathcal{V} \Delta \mathcal{P} = (\mathcal{V} \Delta \mathcal{P}^{C_\mathcal{A}}) \sqcup (\mathcal{V}^{C_\mathcal{A}} \sqcap \mathcal{P}) = \mathcal{H} \sqcup \mathcal{J} = \mathcal{K}$ (say)

Therefore L.H.S = $\mathcal{U} \Delta (\mathcal{V} \Delta \mathcal{P})$ = $\mathcal{U} \Delta \mathcal{K} = (\mathcal{U} \sqcap \mathcal{K}^{C_\mathcal{A}}) \sqcup (\mathcal{U}^{C_\mathcal{A}} \sqcap \mathcal{K})$,Where

$$\mathcal{U} = (U_1, U, \overline{U}(\mu_{1U_1}, \mu_{2U}), L_U)$$

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$$\mathcal{V} = (V_1, V, \bar{V}(\mu_{1V_1,}\mu_{2V}), L_V)$$

$$\mathcal{P} = (P_1, P, \bar{P}(\mu_{1P_1,}\mu_{2P}), L_P)$$

$\mathcal{U}^{C_A} = \mathcal{E} = (E_1, E, \bar{E}(\mu_{1E_1,}\mu_{2E}), L_E)$, where

(i) $E_1 = C_A U_1 - U_1{}^C \sqcup W$ E=U=W where $U^C{}_1 = W_1 - U_1$

(ii) L_E=L_W

(iii)$\mu_{1E_1}: E_1 \rightarrow L_W$ is given by$\mu_{1E_1}x$=M_A

$\mu_{2E}: E \rightarrow L_W$ is given by$\mu_{2E} = \bar{U}x = \mu_{1U_1}x \wedge (\mu_{2U}x)^c$

$\bar{E}: E \rightarrow L_W$ is given by$\bar{E}x = \mu_{1E_1}x \wedge (\mu_{2E}x)^c = M_A \wedge (\bar{U}x)^c$ =$(\bar{U}x)^c$

$\mathcal{V}^{C_A} = \mathcal{F} = (F_1, F, \bar{F}(\mu_{1F_1,}\mu_{2F}), L_F)$, where

(i)$F_1 = C_A V_1 - V_1{}^C \sqcup W$, F=V=W

(ii)L_F=L_V=L_W

(iii) $\mu_{1F_1}: F_1 \rightarrow L_A$ is given by$\mu_{1F_1}x$ =M_A

$\mu_{2F}: F \rightarrow L_W$is given by$\mu_{2F}x = \bar{V}x$

$\bar{F}: F \rightarrow L_W$is given by$\bar{F}x = \mu_{1F_1}x \wedge (\mu_{2F}x)^c = M_A \wedge (\bar{V}x)^c$ =$(\bar{V}x)^c$

$\mathcal{P}^{C_A} = \mathcal{G} = (G_1, G, \bar{G}(\mu_{1G_1,}\mu_{2G}), L_G)$,where

(i)$G_1 = C_A P_1 = P_1{}^C \sqcup W$, G=P=W

(ii)L_G=L_P=L_W

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(iii) $\mu_{1G_1}: G_1 \rightarrow L_W$ is given by $\mu_{1G_1}x = M_A$

$\mu_{2G} : G \rightarrow L_W$ is given by $\mu_{2G}\ x = \bar{P}x$

$\bar{G}: G \rightarrow L_W$ is given by $\bar{G}x = \mu_{1G_1}x \wedge (\mu_{2G}x)^c$

$$= M_A \wedge (\bar{P}x)^c$$

$=(\bar{P}x)^c$

$(\mathcal{V} \sqcap \mathcal{P}^{C_A}) = \mathcal{V} \sqcap \mathcal{G} = \mathcal{H} = (H_1, H, \bar{H}(\mu_{1H_1}, \mu_{2H}), L_H)$, where

(1) $H_1 = V_1 \sqcap G_1 = (V_1 \sqcap P_1{}^C) \sqcup W$, $\mathrm{H} = \mathrm{V} \sqcup \mathrm{G} = \mathrm{W}$

(2) $L_H = L_V \wedge L_G = L_W$

(3) $\mu_{1H_1}: H_1 \rightarrow L_H$ is defined by $\mu_{1H_1}x = \mu_{1V_1}x \wedge \mu_{1G_1}x$

$$= \mu_{1V_1}x \wedge M_A$$

$=\mu_{1V_1}x$

$\mu_{2H} : H \rightarrow L_H$ is defined by $\mu_{2H}x = \mu_{2V}x \vee \mu_{2G}x$

$$= \mu_{2V}x \vee \bar{P}x$$

$\bar{H}: H \rightarrow L_H$ is defined by $\bar{H}x = \mu_{1H_1}x \wedge (\mu_{2H}x)^c$

$$= \mu_{1V_1}x \wedge ((\mu_{2V} \vee \mu_{2G})x)^c$$

$$= \mu_{1V_1}x \wedge (\mu_{2V}x)^c \wedge (\mu_{2G}x)^c$$

$$= (\bar{V}x) \wedge (\mu_{2G}x)^c$$

$$= \bar{V}x \wedge (\bar{P}x)^c$$

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$\mathcal{V}^{C_{\mathcal{A}}} \sqcap \mathcal{P} = \mathcal{F} \sqcap \mathcal{P} = \mathcal{J} = (J_1, J, \bar{J}(\mu_{1J_1,} \mu_{2J}), L_J)$,where

(4)$J_1 = F_1 \sqcap P_1 = ({V_1}^C \sqcup W) \sqcap P_1$

$=({V_1}^C \sqcap P_1) \sqcup (W \sqcap P_1)$

$= ({V_1}^C \sqcap P_1) \sqcup W$, K=F$\sqcup$ P=W$\sqcup$ $W = W$

(5)L_J=$L_F \curlywedge L_P$=L_W

(6)μ_{1J_1}:$J_1 \to L_J$ is defined by $\mu_{1J_1}x = \mu_{1F_1}x \curlywedge \mu_{1P_1}x$

$=M_A \curlywedge \mu_{1P_1}x$

$=\mu_{1P_1}x$

$\mu_{2J} : J \to L_J$ is defined by $\mu_{2J}x = (\mu_{2F} \curlyvee \mu_{2P})x = \mu_{2F}x \curlyvee \mu_{2P}x$

$=\bar{V}x \curlyvee \mu_{2P}x$

$\bar{J}$:$J \to L_J$ is defined by $\bar{J}x = \mu_{1J_1}x \curlywedge (\mu_{2J}x)^c = \mu_{1P_1}x \curlywedge ((\mu_{2F} \curlyvee \mu_{2P})x)^c$

$= \mu_{1P_1}x \curlywedge [(\mu_{2F}x)^c \curlywedge (\mu_{2P}x)^c]$

$= [\mu_{1P_1}x \curlywedge (\mu_{2P}x)^c] \curlywedge (\mu_{2F}x)^c$

$= \bar{P}x \curlywedge (\bar{V}x)^c$

$(\mathcal{V} \sqcap \mathcal{P}^{C_{\mathcal{A}}}) \sqcup (\mathcal{V}^{C_{\mathcal{A}}} \sqcap \mathcal{P}) = \mathcal{H} \sqcup \mathcal{J} = \mathcal{K} = (K_1, K, \bar{K}(\mu_{1K_1,} \mu_{2K}), L_K)$, Where

(7) $K_1 = H_1 \sqcup J_1 = [(V_1 \sqcap {P_1}^C) \sqcup W] \sqcup [({V_1}^C \sqcap P_1) \sqcup W] = (V_1 \Delta\ P_1) \sqcup W$,

$K = H \cap J = W \cap W = W$

(8) $L_K = L_H \vee L_J = L_W$

(9) $\mu_{1K_1} : K_1 \to L_W$ is defined by $\mu_{1K_1} x = (\mu_{1H_1} \vee \mu_{1J_1})x$

Case (i) $x \in W \Rightarrow \mu_{1K_1} x = (\mu_{1H_1} x \vee \mu_{1J_1} x) = \mu_{1V_1} x \vee \mu_{1P_1} x$

Case (ii) $x \notin W$, $x \in V \Rightarrow \mu_{1K_1} x = \mu_{1H_1} x = \mu_{1V_1} x$

Case (iii) $x \notin W$, $x \in P \Rightarrow \mu_{1K_1} x = (\mu_{1J_1} x) = \mu_{1P_1} x$

$\mu_{2K} : W \to L_W$ is defined by $\mu_{2K} x = (\mu_{2H} x \wedge \mu_{2J} x)$

$= (\mu_{2V} x \vee \bar{P}x) \wedge (\bar{V}x \vee \mu_{2P} x)$

$= (\bar{V}x \wedge \bar{P}x) \vee (\mu_{2V} x \wedge \mu_{2P} x)$

$\bar{K} : W \to L_W$ is defined by $\bar{K}x = \mu_{1K_1} x \wedge (\mu_{2K} x)^c$

$= (\mu_{1H_1} \vee \mu_{1J_1})x \wedge (\mu_{2H} x \wedge \mu_{2J} x)^c$

$= (\mu_{1V_1} x \vee \mu_{1P_1} x) \wedge [(\mu_{2V} x \vee \bar{P}x) \wedge (\bar{V}x \vee \mu_{2P} x)^c]$

$= (\mu_{1V} x \vee \mu_{1P_1} x) \wedge [(\bar{V}x \wedge \bar{P}x)^C \vee (\mu_{2V} x \wedge \mu_{2P} x)^c]$

$\mathcal{K}^{C_\mathcal{A}} = \mathcal{L} = (L_1, L, \bar{L}(\mu_{1L_1}, \mu_{2L}), L_L)$, where

(a') $L_1 = C_A K_1 = K_1{}^c \sqcup W$, $\quad L = K = W$

(b') $L_P = L_W$

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(c') $\mu_{1L_1}: L_1 \longrightarrow L_W$ is defined by $\mu_{1L_1}x = M_A$

$\mu_{2L}: L \longrightarrow L_W$ is defined by $\mu_{2L}x = \bar{K}x$

$\bar{L}: L \longrightarrow L_W$, is defined by $\bar{L}x = \mu_{1L_1}x \wedge (\mu_{2L}x)^c = M_A \wedge (\bar{K}x)^c = (\bar{K}x)^c$

$= \{(\mu_{1V_1}x \vee \mu_{1P_1}x) \wedge [(\bar{V}x \wedge \bar{P}x)^C \vee (\mu_{2V}x \wedge \mu_{2P}x)^c]\}^c$

$\mathcal{U} \vartriangle \mathcal{K} = (\mathcal{U} \sqcap \mathcal{K}^{C_{\mathcal{A}}}) \sqcup (\mathcal{U}^{C_{\mathcal{A}}} \sqcap \mathcal{K}) = (\mathcal{U} \sqcap \mathcal{L}) \sqcup (\mathcal{E} \sqcap \mathcal{K})$

$(\mathcal{U} \sqcap \mathcal{K}^{C_{\mathcal{A}}}) = (\mathcal{U} \sqcap \mathcal{L}) = \mathcal{M} = (M, M, \bar{M}(\mu_{1M_1}, \mu_{2M}), L_M)$, where

(10) $M_1 = U_1 \sqcap K_1 = U_1 \sqcap [(V_1 \vartriangle P_1)^C \sqcup W] = [U_1 \sqcap (V_1 \vartriangle P_1)^C] \sqcup (U_1 \sqcap W)$

$= [U_1 \sqcap (V_1 \vartriangle P)^C] \sqcup W$,

$M = U \sqcup K = U \sqcup (H \sqcup J) = U \sqcup (W \sqcup W) = U \sqcup W = W$

(11) $L_M = L_U \wedge L_L = L_W$

(12) $\mu_{1M_1}: M_1 \rightarrow L_W$ is defind by $\mu_{1M_1}x = \mu_{1U_1}x \wedge \mu_{1L_1}x$

$= \mu_{1U_1}x \wedge M_A$

$= \mu_{1U_1}x$

$\mu_{2M}: M \rightarrow L_W$ is defined by $\mu_{2M}x = (\mu_{2U} \vee \mu_{2L})x$

$= (\mu_{2U}x \vee \mu_{2L}x)$

$= \mu_{2U}x \vee \bar{K}x$

$=\mu_{2U}x \curlyvee [\mu_{1K_1}x \curlyvee (\mu_{2K}x)^c]$

$= \mu_{2U}x \curlyvee [(\mu_{1V_1}x \curlyvee \mu_{1P_1}x) \curlywedge [(\mu_{2V}x \curlyvee \bar{P}x) \curlywedge (\bar{V}x \curlyvee$

$\mu_{2P}x)]^c]$

$\bar{M}:M \rightarrow L_W$ is defined by $\bar{M}x = \mu_{1M_1}x \curlywedge (\mu_{2M}x)^c$

$=\mu_{1U_1}x \curlywedge [\mu_{2U}x \curlyvee [(\mu_{1V_1}x \curlyvee \mu_{1P_1}x) \curlywedge [(\mu_{2V}x \curlyvee \bar{P}x) \curlywedge (\bar{V}x \curlyvee$

$\mu_{2P}x)]^c]$

$(\mathcal{U}^{c_A} \sqcap \mathcal{K})=(\mathcal{E} \sqcap \mathcal{K})=\mathcal{N} = (N_1, N, \bar{N}(\mu_{1N_1}\mu_{2N}), L_N)$, where

(1a) $N_1 = E_1 \sqcap K_1=[(U_1)^C \sqcup W] \sqcap (K_1 \sqcup W) = [[U_1{}^C \sqcup W)] \sqcap$

$(V_1 \triangle P_1) \sqcup W]$

$=[U_1{}^C \sqcap (V_1 \triangle P_1)] \sqcup W$

(1b) $L_N = L_E \sqcap L_K = L_W$

(1c) $\mu_{1N_1}: L_N \rightarrow L_W$ is defind by $\mu_{1N_1}x = \mu_{1E_1}x \curlywedge \mu_{1K_1}x = M_A \sqcap$

$(\mu_{1H_1} \curlyvee \mu_{1J_1})x$

$= (\mu_{1H_1} \curlyvee \mu_{1J_1})x$

$=(\mu_{1V_1} \curlyvee \mu_{1P_1})x$

$\mu_{2N}: N \rightarrow L_W$ is defined by $\mu_{2N}x=(\mu_{2E} \curlyvee \mu_{2K})x$

$=(\bar{U}x \curlyvee \mu_{2K}x)$

$=\bar{U}x \curlyvee [(\mu_{2V}x \curlyvee \bar{P}x) \curlywedge (\bar{V}x \curlyvee$

$\mu_{2P}x)]$

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$= [(\mu_{2V}x \curlyvee \bar{P}x \curlyvee \bar{U}x) \curlywedge (\bar{V}x \curlyvee \mu_{2P}x \curlyvee \bar{U}x)]$

$\bar{N}: N \to L_W$ is defined by $\bar{N}x = \mu_{1N_1}x \curlywedge (\mu_{2N}x)^c$

$= \mu_{1U_1}x \curlywedge [\mu_{2U}x \curlyvee [(\mu_{1V_1}x \curlyvee \mu_{1P_1}x) \curlywedge [(\mu_{2V}x \curlyvee \bar{P}x) \curlywedge (\bar{V}x \curlyvee \mu_{2P}x)]^C]$

$\mathcal{U}\Delta(\mathcal{V}\Delta\mathcal{P}) = \mathcal{U}\Delta\mathcal{K} = (\mathcal{U} \sqcap \mathcal{K}^{C_\mathcal{A}}) \sqcup (\mathcal{U}^{C_\mathcal{A}} \sqcap \mathcal{K}) = \mathcal{M} \sqcup \mathcal{N}$

$= \mathcal{Q} = (Q_1, Q, \bar{Q}(\mu_{1Q_1}, \mu_{2Q}), L_Q)$, where

(1) $Q_1 = M_1 \sqcup N_1 = [(U_1 \sqcap (V_1 \Delta P_1)^C) \sqcup W] \sqcup [(U_1{}^C \sqcap (V_1 \vartriangle P_1)) \sqcup W]$

$= [U_1 \vartriangle (V_1 \vartriangle P_1)] \sqcup W$, Q=M ⊓N =W

(2) $L_Q = L_M \sqcap L_N = L_W$

(3) (3)$\mu_{1Q_1}: Q_1 \to L_W$ is defind by $\mu_{1Q_1}x = (\mu_{1M_1} \curlyvee \mu_{1N_1})$ x

Case (i) x∈W $\Rightarrow \mu_{1Q_1}x = (\mu_{1M_1}x \curlyvee \mu_{1N_1}x) = \mu_{1U_1}x \curlyvee \mu_{1V_1}x \curlyvee \mu_{1P_1}x$

Case (ii) x$\notin W$, x∈U $\Rightarrow \mu_{1Q_1}x = \mu_{1M_1}x = \mu_{1U_1}x$

Case (iii) x$\notin W$, x∈V $\Rightarrow \mu_{1Q_1}x = (\mu_{1N_1}x) = \mu_{1V_1}x$

Case (iii) x$\notin W$, x∈P $\Rightarrow \mu_{1Q_1}x = (\mu_{1N_1}x) = \mu_{1P_1}x$

$\mu_{2Q}: Q \to L_W$ is defined by $\mu_{2Q}x = (\mu_{2M}x \curlywedge \mu_{2N}x)$

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$= \quad [\mu_{2U}x \curlyvee [(\mu_{1V_1}x \curlyvee \mu_{1P_1}x) \curlywedge ((\bar{V}x)^C \curlyvee (\bar{P}x)^C)) \curlywedge ((\mu_{2V}x)^C \curlyvee (\mu_{2P}x)^C)]] \curlywedge [(\mu_{2V}x \curlyvee \bar{P}x \curlyvee \bar{U}x) \curlywedge (\bar{V}x \curlyvee \mu_{2P}x \curlyvee \bar{U}x)]$

$= (\mu_{1V_1}x \curlyvee \mu_{1P_1}x \curlyvee \mu_{2U}x) \curlywedge \{(\bar{V}x)^C \curlyvee (\bar{P}x)^C) \curlyvee \mu_{2U}x\} \curlywedge (\mu_{2Vx} \curlyvee \bar{P}x \curlyvee \bar{U}x)$

$\curlywedge (\bar{V}x \curlyvee \mu_{2P}{}^x \curlyvee \bar{U}x)\{(\mu_{2V}x)^C \curlyvee (\mu_{2P}x)^C \curlyvee \mu_{2U}x\}$

$\mathcal{U} \sqcap \mathcal{F} \;=\; \mathcal{R} = (R_1, R, \bar{R}(\mu_{1R_1,}\mu_{2R}), L_R)$,where

(4) $R_1 = U_1 \sqcap F_{1=}U_1 \sqcap [V_1{}^C \sqcup W] = [U_1 \sqcap V_1{}^C] \sqcup W$

R=U⊔ F =W⊔ W=W

(5) $L_R = L_U \curlywedge L_F = L_W$

(6) $\mu_{1R_1}: R_1 \to L_W$ is defind by $\mu_{1R_1}x = \mu_{1U_1}x \curlywedge \mu_{1F_1}x$

$= \mu_{1U_1}x \curlywedge M_A$

$= \mu_{1U_1}x$

μ_{2R}: R→ L_W is defined by $\mu_{2R}x = (\mu_{2U} \curlyvee \mu_{2F})$x

$= (\mu_{2U}x \curlyvee \mu_{2F}x)$

$= \mu_{2U}x \curlyvee \bar{V}$x

$\bar{R}: R \to L_W$ is defined by $\bar{R}x = \mu_{1R_1}x \curlywedge (\mu_{2R}x)^c = \mu_{1U_1}x \curlywedge [(\mu_{2U}x) \curlyvee \bar{V}x)]^c$

$= \bar{U}x \curlywedge (\bar{V}x)^c$

$(\mathcal{U}^{C_A} \sqcap \mathcal{V}) = \mathcal{E} \sqcap \mathcal{V} = \mathcal{S} = (S_1, S, \bar{S}(\mu_{1S_1}, \mu_{2S}), L_S)$, where

(7)$S_1 = E_1 \sqcap V_1 = [U_1{}^C \sqcup W] \sqcap V_1 = [U_1{}^C \sqcup V_1] \sqcup W$, S=E⊔V=W

(8) $L_S = L_E \wedge L_V = W$

(9) $\mu_{1S_1} : S_1 \to L_W$ is defined by $\mu_{1S_1}x = \mu_{1E_1}x \wedge \mu_{1V_1}x$

$= \mu_{1V_1}x \wedge M_A$

$= \mu_{1V_1}x$

$\mu_{2S}: S \to L_W$ is defined by $\mu_{2S}x = (\mu_{2E} \vee \mu_{2V})x$

$= (\mu_{2E}x \vee \mu_{2V}x)$

$= \mu_{2V}x \vee \bar{U}x$

$\bar{S}: S \to L_W$ is defined by $\bar{S}x = \mu_{1S_1}x \wedge (\mu_{2S}x)^c$

$= \mu_{1V_1}x \wedge [(\mu_{2V}x) \vee \bar{U}x)]^c$

$= \mu_{1V_1}x \wedge (\bar{U}x)^C \wedge (\mu_{2V}x)^C$

$= \bar{V}x \wedge (\bar{U}x)^c$

$\mathcal{R} \sqcup \mathcal{S} = \mathcal{T} = (T_1, T, \bar{T}(\mu_{1T_1}, \mu_{2T}), L_T)$, where

(10)$T_1 = R_1 \sqcup S_1 = \left[[U_1 \sqcap V_1{}^C] \sqcup W\right] \sqcup \left[[(U_1{}^C \sqcap V_1)] \sqcup W\right]$

$= (U_1 \Delta V_1) \sqcup W$, T=R⊓S=W

(11) $L_T = L_R \wedge L_S = L_W$

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(12) $\mu_{1T_1}: T_1 \to L_W$ is defined by $\mu_{1T_1}x = (\mu_{1R_1} \curlyvee \mu_{1S_1})x$

$= (\mu_{1U_1} \curlyvee \mu_{1V_1})x$

Case (i) $x \in W$ $\mu_{1T_1}x = (\mu_{1U_1}x \curlyvee \mu_{1V_1}x)$

$= (\mu_{1U_1}x \curlyvee \mu_{1V_1}x)$

Case (ii) $x \notin W$, $x \in U$ $\mu_{1T_1}x = (\mu_{1U_1}x)$

Case (iii) $x \notin W$, $x \in U \mu_{1T_1}x = \mu_{1C_1}x$

$\mu_{2T}: T \to L_W$ is defined by $\mu_{2T}x = (\mu_{2R} \curlyvee \mu_{2S})x$

$= (\mu_{2U}x \curlyvee \bar{V}x) \curlywedge (\bar{U}x \curlyvee \mu_{2V}x)$

$= (\bar{V}x \curlywedge \bar{U}x) \curlyvee (\mu_{2U} \curlywedge \mu_{2V}x)$

$\bar{T}: T \to L_W$ is defined by $\bar{T}x = \mu_{1T_1}x \curlywedge (\mu_{2T}x)^c$

$= \mu_{1T_1}x \curlywedge [(\bar{V}x \curlywedge \bar{U}x) \curlyvee (\mu_{2U} \curlywedge \mu_{2V}x)]^c$

$= (\mu_{1R_1}x \curlyvee \mu_{1S_1}x) \curlywedge ((\bar{V}x \curlywedge \bar{U}x)^C) \curlyvee (\mu_{2U} \curlywedge \mu_{2V}x)^C$

$= (\mu_{1U_1}x \curlyvee \mu_{1V_1}x) \curlywedge (\bar{V}x \curlywedge \bar{U}x)^C) \curlywedge (\mu_{2U}x \curlywedge \mu_{2V}x)^C$

$\mathcal{T}^{C_{\mathcal{A}}} = \mathcal{X} = (X_1, X, \bar{X}(\mu_{1X_1}, \mu_{2X}), L_X)$, where

(a') $X_1 = C_A T_1 = T_1^C \sqcup W = [(U_1 \vartriangle V_1)^C \sqcup W]$, $X = T = W$

(b') $L_X = L_T = L_W$

(c') $\mu_{1X_1}: T_1 \longrightarrow L_W$ is defined by $\mu_{1X_1}x = M_A$

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$\mu_{2X}: X \longrightarrow L_W$ is defined by $\mu_{2X}x = \overline{T}x = \mu_{1T_1}x \wedge (\mu_{2T}x)^C$

$\overline{X}: X \longrightarrow L_W$ is defined by $\overline{X}x = \mu_{1X_1}x \wedge (\mu_{2X}x)^C$

$$= M_A \wedge (\overline{T}x)^C = (\overline{T}x)^C$$

$$(\mu_{1U_1}x \vee \mu_{1V_1}x) \wedge [(\bar{V}x \wedge \bar{U}x)^c \vee (\mu_{2U} \wedge \mu_{2V}x)^c]\}^C$$

$(\mathcal{T} \sqcap \mathcal{P}^{C_\mathcal{A}}) = (\mathcal{T} \sqcap \mathcal{G}) = \mathcal{A} = (A_1, A, \bar{A}(\mu_{1A_1}, \mu_{2A}), L_A)$, where

(13) $A_1 = T_1 \sqcap G_1 = [(V_1 \vartriangle U_1) \sqcap P_1{}^C \sqcup W]$, V=T$\sqcup$ G=W

(14) $L_A = L_T \vee L_G = L_W$

(15) $\mu_{1A_1}: A_1 \rightarrow L_W$ is defined by $\mu_{1A_1}x = \mu_{1T_1}x \wedge \mu_{1G_1}x$

$$= (\mu_{1U_1} \vee \mu_{1V_1})x \wedge M_A$$

$$= (\mu_{1U_1} \vee \mu_{1V_1})x$$

$\mu_{2A}: A \rightarrow L_W$ is defined by $\mu_{2A}x = (\mu_{2T} \vee \mu_{2G})x$

$$= (\mu_{2T}x \vee \mu_{2G}x)$$

$$= \mu_{2T}x \vee \bar{P}x$$

$$= [(\mu_{2V}x \vee \bar{P} \vee \bar{U}x) \wedge (\bar{V}x \vee \mu_{2U}x \vee \bar{P})] \vee \bar{P}x$$

$$= [(\mu_{2V}x \vee \bar{U}x \vee \bar{P}x) \wedge (\bar{V}x \vee \mu_{2U}x \vee \bar{P}x)]$$

$\bar{A}: A \rightarrow L_W$ is defined by $\bar{A}x = \mu_{1A_1}x \wedge (\mu_{2A}x)^C$

$$= (\mu_{1U_1} \vee \mu_{1V_1})x \wedge [(\mu_{2V}x \vee \bar{U}x \vee \bar{P}x) \wedge (\bar{V}x \vee \mu_{2U}x \vee \bar{P}x)]^C$$

$(\mathcal{T}^{C_\mathcal{A}} \sqcap \mathcal{P}) = \mathcal{X} \sqcap \mathcal{P} = \mathcal{Y} = (Y_1, Y, \bar{Y}(\mu_{1Y_1}, \mu_{2Y}), L_Y)$, where

...rests upon the authors

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(16)$Y_1 = X_1 \sqcap P_{1=}[X_1{}^C \sqcap P_1] = [(U_1 \Delta V_1)^C \sqcap P_1] \sqcup W$, Y=U⊔ P =W

(17) $L_Y = L_X \wedge L_P = L_W$

(18) $\mu_{1Y_1}: Y_1 \to L_W$ is defined by $\mu_{1Y_1}x = \mu_{1X_1}x \wedge \mu_{1P_1}x$

$=\mu_{1P_1}x \wedge M_A$

$=\mu_{1P_1}x \wedge M_A$

$=\mu_{1P_1}x$

$\mu_{2Y}: Y \to L_W$ is defined by $\mu_{2Y}x=(\mu_{2X} \curlyvee \mu_{2P})$x

$=(\mu_{2X}x \curlyvee \mu_{2P}x)$

$=[(\mu_{1U_1}x \wedge \mu_{1V_1}x) \wedge (\bar{V}x \wedge \bar{U}x)^C \wedge (\mu_{2U}x \wedge$
$\mu_{2V}x)^C] \curlyvee \mu_{2P}x$

$\bar{Y}$:$Y \to L_W$ is defined by $\bar{Y}x = \mu_{1Y_1}x \wedge (\mu_{2Y}x)^c$

$= \mu_{1P_1}x \wedge [(\mu_{1U_1}x \wedge \mu_{1V_1}x) \wedge (\bar{V}x \wedge \bar{U}x)^C \wedge (\mu_{2U}x \wedge \mu_{2V}x)^C]^c$

$= \mu_{1P_1}x \wedge \left[(\mu_{1U_1}x)^C \curlyvee (\mu_{1V_1}x)^C\right] \curlyvee (\bar{V}x \wedge \bar{U}x) \curlyvee$
$(\mu_{2U}x \wedge \mu_{2V}x)$

$(\mathcal{U} \Delta \mathcal{V}) \Delta \mathcal{P} = \mathcal{R} \Delta \mathcal{P} = (\mathcal{R} \sqcap \mathcal{P}^{C_\mathcal{A}}) \sqcup (\mathcal{R}^{C_\mathcal{A}} \sqcap \mathcal{P})=$ $\mathcal{A} \sqcup \mathcal{Y} = \mathcal{Z} =$
$(Z_1, Z, \bar{Z}(\mu_{1Z_1,}\mu_{2Z}), L_Z)$ where

(2a) $Z_1 = A_1 \sqcup Y_{1=}[(U_1 \Delta V_1) \sqcap P_1{}^C) \sqcup W] \sqcup [(U_1 \Delta V_1)^C \sqcap P_1) \sqcup$ W]

$=[U_1 \Delta (V_1 \Delta P_1)] \sqcup$ W , Z=A ⊓Y =W

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(2b) $L_Z = L_A \sqcap L_Y = L_W$

(2c) $\mu_{1Z_1} : Z_1 \to L_W$ is defind by $\mu_{1Z_1} x = (\mu_{1A_1} \curlyvee \mu_{1Y_1})\, x$

Case (i) x∈A $\Rightarrow \mu_{1Z_1} x = (\mu_{1A_1} x \curlyvee \mu_{1Y_1} x)$

$$= \mu_{1U_1} x \curlyvee \mu_{1V_1} x \curlyvee \mu_{1W_1} x$$

Case (ii) $x \notin W$, x∈U $\Rightarrow \mu_{1Z_1} x = (\mu_{1A_1} x) = \mu_{1U_1} x$

Case (iii) $x \notin W$, x∈V $\Rightarrow \mu_{1Z_1} x = (\mu_{1A_1} x) = \mu_{1V_1} x$

Case (iii) $x \notin W$, x∈P $\Rightarrow \mu_{1Z_1} x = (\mu_{1Y_1} x) = \mu_{1P_1} x$

$\mu_{2Z} : W \to L_W$ is defined by $\mu_{2Z} x = \mu_{2A} x \curlywedge \mu_{2Y} x$

$= [(\mu_{2U} x \curlyvee \bar{P}x \curlyvee \bar{V}x) \curlywedge (\mu_{2V} x \curlyvee \bar{P}x \curlyvee \bar{U}x)] \curlywedge [(\mu_{1V_1} x \curlyvee \mu_{1U_1} x \curlyvee \mu_{2P} x) \curlywedge \{((\bar{V}x)^C \curlyvee (\bar{U}x)^C) \curlyvee \mu_{2P} x\} \curlywedge \{(\mu_{2V} x)^C \curlyvee (\mu_{2U} x)^C \curlyvee \mu_{2P} x\}]$

$= (\mu_{2U} x \curlyvee \bar{P}x \curlyvee \bar{V}x) \curlywedge (\mu_{2V} x \curlyvee \bar{P}x \curlyvee \bar{U}x) \curlywedge (\mu_{1V_1} x \curlyvee \mu_{1U_1} x \curlyvee \mu_{2P} x) \curlywedge$

$\{((\bar{V}x)^C \curlyvee (\bar{U}x)^C) \curlyvee \mu_{2P} x\} \curlywedge \{(\mu_{2V} x)^C \curlyvee (\mu_{2U} x)^C \curlyvee \mu_{2P} x\}$

$\bar{Z} : Z \to L_W$ is defined by $\bar{Z}x = \mu_{1Z_1} x \curlywedge (\mu_{2Z}\, x)^c$

$$= (\mu_{1A_1} \curlyvee \mu_{1Y_1}) x \curlywedge (\mu_{2A} x \curlywedge \mu_{2Y} x)^c$$

$= (\mu_{1U_1} x \curlyvee \mu_{1V_1} x \curlyvee \mu_{1P_1} x) \curlywedge [(\mu_{2U} x \curlyvee \bar{P}x \curlyvee \bar{V}x) \curlywedge (\mu_{2V} x \curlyvee \bar{P}x \curlyvee \bar{U}x) \curlywedge (\mu_{1V_1} x \curlyvee \mu_{1U_1} x \curlyvee \mu_{2P} x) \curlywedge \{((\bar{V}x)^C \curlyvee (\bar{U}x)^C) \curlyvee \mu_{2P} x\} \curlywedge \{(\mu_{2V} x)^C \curlyvee (\mu_{2U} x)^C \curlyvee \mu_{2P} x\}]^c$

Need to show Q and Z are full equal i.e. sufficient to show

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(I) $Q_1 = Z_1$,

(II) $L_Q = L_Z$

(III) $\mu_{1Q_1}x = \mu_{1Z_1}x$ and

The minimum of 1,2,3,4and 5 and the minimum of 1',2',3',4' and 5' are same using the given conditions ,So that $\mu_{2Q}x = \mu_{2Z}x$ are equal

(I) follows from a (1a)and(2a)

(II) follows from a(1b)and(2b)

(III) follows from a(1c)and(2c)

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