Implementation of Unscented Kalman Filter to Autonomous Aerial Vehicle for Target Tracking

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***Abstract*—Unscented Kalman filter is applied for tracking of a 3-dimensional autonomous aerial target. The noise corrupted measurements are smoothed and at the same time the vehicle's velocity components are found out. Detailed study is carried out in Monte-Carlo simulation. The outputs of the algorithm are compare with that of extended Kalman filter and are useful for releasing weapon on to the target.**

Keywords—3-Dimensional tracking, Autonomous aerial vehicle, Estimation, Study on Target Motion, Target Tracking, Unscented Kalman Filter

I. Introduction

Autonomous Aerial Vehicle (AAV) is the world's most reliable airborne warfare system today. AAV is an on-board computer controlled and piloted robotic system that can be maneuvered in 3-dimensions [1-2]. AAV, under the most environmental situation, has access to follow pre-programmed path wherever and whenever needed. It transmits radio frequency signals to track the target vehicle factors like range, azimuth bearing and elevation. Recent AAVs have satellite-based transmissions systems offering the capability to check and redirect AAV assignments worldwide from a ship or from helicopter [2-3]. Due to this reason, semi-autonomous procedures are advantageous over fully autonomous processes. Weapon release system can be a ship on the water surface or a helicopter or an aircraft in air. AAV Data obtained is transmitted to weapon release system through GPS such that weapon release system will be capable to know the status and movement of target and emits weapon in that course. Tracking of target is conducted using Unscented Kalman filter (UKF) [4-5]. Target movement parameters specifically at extended distances are nonlinear. So, UKF is thought built on quickly convergent and unbiased filter challenges in extended Kalman filter and Kalman filter [6-8].

The movement of target is a non-linear process as the relation of azimuth angle and elevation measurements with the state of target vector is nonlinear [7-8]. Hence the optimal Kalman filter [9-10] is not proposed. Extended Kalman Filter (EKF) is a suboptimal nonlinear filter that operates by linearizing the nonlinearities of the system. Linearization of the nonlinearities results in loss of information while estimating the target parameters. The target moves at constant course, pitch and speed. The noise of the process is considered to be white Gaussian.

Mathematical modelling of UKF and EKF algorithms with simulator is given in Section II. Section III deals about the simulation results are elaborated in Section III and then concluded in section IV.

II. Mathematical Modelling

First, consider state vector:

$$X_S(j) = \begin{bmatrix} \dot{x}(g) \\ \dot{y}(g) \\ \dot{z}(g) \\ R_x(g) \\ R_y(g) \\ R_z(g) \end{bmatrix} \quad (1)$$

Here $\dot{x}(g), \dot{y}(g), \dot{z}(g)$ are g target velocity components and $R_x(g), R_y(g), R_z(g)$ are target range elements in x, y and z directions respectively [7-10]. The state vector up-dation becomes:

$$X_s(g+1) = X_s(g)\emptyset + b(g+1) + \Gamma w(g) \quad (2)$$

$\emptyset$ is given by:

$$\emptyset = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ t & 0 & 0 & 1 & 0 & 0 \\ 0 & t & 0 & 0 & 1 & 0 \\ 0 & 0 & t & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

The time interval at which measurements are collected is t. The determinist control matrix is $b(g+1)$, given by:

$$b(g+1) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ -(x_0(g+1) - x_0(g)) \\ -(y_0(g+1) - y_0(g)) \\ -(z_0(g+1) - z_0(g)) \end{bmatrix}^T \quad (4)$$

Here $x_0(g), y_0(g)\ and\ z_0(g)$ are components of observer location in x, y and z directions respectively. To diminish the mathematical complication, true North convention is followed by all angles. Let $w(g)$ be plant noise.

$$w(g) = [w_x \quad w_y \quad w_z]^T \quad (5)$$

Variance of $w(g)$ is given by:

$$E[\Gamma(g) w(g) w^T(g) \Gamma^T(g)] = S_{ij} \quad (6)$$

Where $$\delta_{ij} = \sigma_w^2 \ (i = j) \quad (7)$$

$= 0$ otherwise

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$$S = \begin{bmatrix} ts^2 & 0 & 0 & ts^3/2 & 0 & 0 \\ 0 & ts^2 & 0 & 0 & ts^3/2 & 0 \\ 0 & 0 & ts^2 & 0 & 0 & ts^3/2 \\ ts^3/2 & 0 & 0 & ts^3/4 & 0 & 0 \\ 0 & ts^2/2 & 0 & 0 & ts^3/4 & 0 \\ 0 & 0 & ts^2/2 & 0 & 0 & ts^3/4 \end{bmatrix} \quad (8)$$

$$\Gamma(\mathscr{g}) = \begin{bmatrix} t & 0 & 0 \\ 0 & t & 0 \\ 0 & 0 & t \\ t^2/2 & 0 & 0 \\ 0 & t^2/2 & 0 \\ 0 & 0 & t^2/2 \end{bmatrix} \quad (9)$$

$Z(\mathscr{g})$ is the measurement matrix and is given by:

$$Z(\mathscr{g}) = [R_m(\mathscr{g}) \quad B_m(\mathscr{g}) \quad \Theta_m(\mathscr{g})]^T \quad (10)$$

Here $R_m(\mathscr{g})$, $B_m(\mathscr{g})$ and $\Theta_m(\mathscr{g})$ are measured range, bearing and elevation.

$$R_m(\mathscr{g}) = R(\mathscr{g}) + \xi_R(\mathscr{g}) \quad (11)$$

$$B_m(\mathscr{g}) = B(\mathscr{g}) + \xi_B(\mathscr{g}) \quad (12)$$

$$\Theta_m(\mathscr{g}) = \Theta(\mathscr{g}) + \xi_\Theta(\mathscr{g}) \quad (13)$$

where $R(\mathscr{g}), B(\mathscr{g})$ and $E(\mathscr{g})$ are true range, true bearing and true elevation.

$$R(\mathscr{g}) = \sqrt{R_x^2(\mathscr{g}) + R_y^2(\mathscr{g}) + R_Z^2(\mathscr{g})} \quad (14)$$

$$B(\mathscr{g}) = tan^{-1}\big(R_x(\mathscr{g})/R_y(\mathscr{g})\big) \quad (15)$$

$$\Theta(\mathscr{g}) = tan^{-1}\big(R_{xy}(\mathscr{g})/R_Z(\mathscr{g})\big) \quad (16)$$

$$\text{Where } R_{xy} = \sqrt{R_x^2 + R_y^2} \quad (17)$$

Measurement vector is given by

$$Z(\mathscr{g}) = H(\mathscr{g})X_s(\mathscr{g}) + \xi(\mathscr{g}) \quad (18)$$

$$H(\mathscr{g}) = \begin{bmatrix} 0 & 0 & 0 & \sin(\hat{B})\sin(\hat{\Theta}) & \sin(\hat{\Theta})\cos(\hat{B}) & \cos(\hat{\Theta}) \\ 0 & 0 & 0 & \cos(\hat{B})/R_{xy} & -\sin(\hat{B})/R_{xy} & 0 \\ 0 & 0 & 0 & \sin(\hat{B})\cos(\hat{\Theta})/R & \cos(\hat{\Theta})\cos(\hat{B})/R & -\sin(\hat{\Theta})/R \end{bmatrix} \quad (19)$$

$$\xi(\mathscr{g}) = [\, \xi_R \quad \xi_B \quad \xi_\Theta \,]^T \quad (20)$$

The unscented Kalman filter is a combination of classical filter and an unscented transformation, which is made in order to transmit transformation in the model through a non-linear process. UKF gives adequately precise solution.

An easy method is adapted to evaluate the statistical properties of a random variable, which endures a non-linear transformation is called an unscented transformation. Suppose a random variable x, having an expected value $\bar{x}$, covariance P_x and dimension Λ, imparting through $y = g(x)$. $2\Lambda + 1$ sigma vectors are used to compute statistics of y as follows:

$$\chi_0 = \bar{x}$$

$$\chi_a = \bar{x} + \left(\sqrt{(\Lambda + \lambda) + P_x}\right)_a \quad a = 1,2,\ldots\ldots,\Lambda$$

$$\chi_a = \bar{x} - \left(\sqrt{(\Lambda + \lambda) + P_x}\right)_{a-\Lambda} \quad a = \Lambda + 1,\ldots\ldots,2\Lambda$$

$$W_0^{(m)} = \lambda/(\Lambda + \lambda) \quad (21)$$

$$W_0^{(c)} = \lambda/(\Lambda + \lambda) + (1 - \alpha^2 + \beta)$$

$$W_a^{(m)} = W_a^{(c)} = 1/\big(2(\Lambda + \lambda)\big) \qquad a = 1,2,\ldots,2\Lambda$$

Here $\lambda = \alpha^2(\Lambda + \kappa) - \Lambda$ is a parameter for scaling. α is chosen to be small definite positive value, say 0.001, and defines how the sigma values are distributed over the mean. κ, a tuning parameter is chosen as zero. β integrates earlier information of the distribution of x (for Gaussian density function, $\beta = 2$ is the best fit). i^{th} row of the matrix root is represented as $\left(\sqrt{(\Lambda + \lambda) + P_x}\right)_a$. $W_0^{(m)}, W_0^{(c)}$, $W^{(m)}$ and $W^{(c)}$ characterizes the weights of primed target vector, its covariance, sigma point matrix and its covariance matrix respectively [7-10]. Equation (22) represents the non-linear function utilised to propagate sigma vectors

$$y_a = g(\chi_a) \qquad a = 1,2,\ldots.,2\Lambda \quad (22)$$

The weighted posterior sigma points mean, and covariance are utilised to predict the covariance and mean of x [13].

$$\bar{y} \approx \textstyle\sum_{a=0}^{2\Lambda} W_a^{(m)} y_a \quad (23)$$

$$P_y \approx \textstyle\sum_{a=0}^{2\Lambda} W_a^{(c)} \{y_a - \bar{y}_a\}\{y_a - \bar{y}_a\}^T \quad (24)$$

UKF implementation is as follows.

(1). Let Λ be the vector of target state dimensions. $(2\Lambda + 1)$ state vectors are computed using sigma points from the initial points

$$X(\mathscr{g}) = \begin{bmatrix} X_s(\mathscr{g}) \\ X_s(\mathscr{g}) + \sqrt{(\Lambda + \lambda) + P(\mathscr{g})} \\ X_s(\mathscr{g}) - \sqrt{(\Lambda + \lambda) + P(\mathscr{g})} \end{bmatrix}^T \quad (25)$$

(2). Based on the process model (2), transform the sigma points.

(3). The estimate of the predicted state at the time $(m + 1)$ of m observations is given as

$$X_s(\mathscr{g} + 1) = \textstyle\sum_{a=0}^{2n} W_a^{(m)} X_s\big(a, (\mathscr{g} + 1)\big) \quad (26)$$

(4). Considering additive and independent process noise, the estimated covariance matrix is taken as:

$$P(\mathscr{g} + 1) = \textstyle\sum_{a=0}^{2n} W_a^{(c)} \big[-X_s(\mathscr{g} + 1) + X_s\big(a, (\mathscr{g} + 1)\big)\big]\big[-X_s(\mathscr{g} + 1) + X_s\big(a, (\mathscr{g} + 1)\big)\big]^T + Q(\mathscr{g}) \quad (27)$$

(5). The sigma points are modified using the average and the covariance predicted as follows

$$X(\mathscr{g} + 1) = \begin{bmatrix} X_s(\mathscr{g} + 1) \\ X_s(\mathscr{g} + 1) + \sqrt{(\Lambda + \lambda) + P(\mathscr{g} + 1)} \\ X_s(\mathscr{g} + 1) - \sqrt{(\Lambda + \lambda) + P(\mathscr{g} + 1)} \end{bmatrix}^T \quad (28)$$

(6). based on the measurement model given in (16), transform the expected sigma points. The matrix of estimated measurements is

$$\hat{z}(\mathscr{g} + 1) = \textstyle\sum_{a=0}^{2\Lambda} W_a^{(m)} Y(\mathscr{g} + 1) \quad (29)$$

$$Y(m + 1) = h\big(X_s(\mathscr{g} + 1)\big) \quad (30)$$

(7). The matrix of covariances for innovation is determined as

$$P_{yy} = \sum_{a=0}^{2\Lambda} W_a^{(c)} \left[-\hat{z}(g+1) + Y\left(a,(g+1)\right)\right]\left[-\hat{z}(g+1) + Y\left(a,(g+1)\right)\right]^T + \sigma_B^2(g) \quad (31)$$

(8). The cross-covariance matrix is calculated as

$$P_{xy} = \sum_{a=0}^{2\Lambda} W_a^{(c)} \left[-X_s(g+1) + X_s\left(a,(g+1)\right)\right]\left[-X_s(g+1) + X_s\left(a,(g+1)\right)\right]^T \quad (32)$$

(9). Kalman gain is calculated as

$$G(g+1) = P_{xy} P_{yy}^{-1} \quad (33)$$

(10). The estimated state is calculated as

$$X(g+1) = \left(\hat{z}(g+1) - \hat{z}(g+1)\right) * \left(X(g+1) + G(g+1)\right) \quad (34)$$

where $z(g+1)$ is measurement vector matrix.

(11). The error in estimated covariance matrix is

$$P(g+1) = -G(g+1) P_{yy} G^T(g+1) + P(g+1) \quad (35).$$

Initial target state vector, target velocity components are computed using first and second measurement sets of range bearing and elevation measurements [7-10].

Let the opening position of observer and target be (x_0, y_0, z_0) and (x_t, y_t, z_t) assuming that they travel with velocities v_t and v_0. After t seconds, shift in observer position is given as follows

$$dx_0 = v_0 * \sin(ocr) * \sin(oph) * t \quad (36)$$

$$dy_0 = v_0 * \cos(ocr) * \sin(oph) * t \quad (37)$$

$$dz_0 = v_0 * \cos(oph) * t$$

Here oph and ocr are observer pitch and observer course respectively. The modified location of observer is presented as follows.

$$x_0 = x_0 + dx_0 \quad (38)$$

$$y_0 = y_0 + dy_0 \quad (39)$$

$$z_0 = z_0 + dz_0 \quad (40)$$

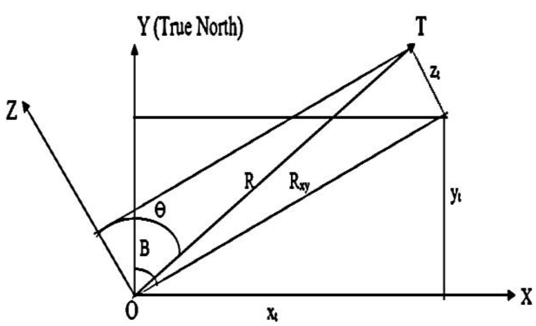


Fig. 1Positions of target and observer assumed

Correspondingly, as of Fig.1.

$$x_t = R_{xy} * \sin(B) \quad (41)$$

$$y_t = R_{xy} * \cos(B) \quad (42)$$

$$\sin(\Theta) = R_{xy}/R \quad (43)$$

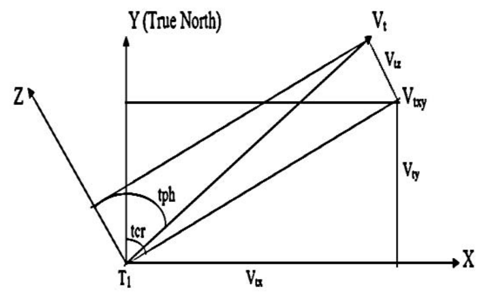
Substituting (43) in (41) and (42),

$$x_t = R * \sin(\Theta) * sin(B) \quad (44)$$

$$y_t = R * \sin(\Theta) * cos(B) \quad (45)$$

$$z_t = R * cos(\Theta) \quad (46)$$

When the target is in motion with velocity v_t, the target position changes after t seconds, as shown in Fig.2.



$$dx_t = v_t * \sin(tcr) * \sin(tph) * t \quad (47)$$

Fig. 2 Velocity components of target and observer

$$dy_t = v_t * \cos(tcr) * \sin(tph) * t \quad (48)$$

$$dz_t = v_t * \cos(tph) * t \quad (49)$$

Here tph and tcr are target pitch and target course respectively.

The modified location of target is presented as follows.

$$x_t = x_t + dx_t \quad (50)$$

$$y_t = y_t + dy_t \quad (51)$$

$$z_t = z_t + dz_t \quad (52)$$

Computer-generated actual values of azimuth bearing, range and elevation are calculated as follows.

$$true\ bearing = tan^{-1}((x_t - x_0)/(y_t - y_0)) \quad (53)$$

$$true\ range = \sqrt{(x_t - x_0)^2 + (y_t - y_0)^2 + (z_t - z_0)^2} \quad (54)$$

$$true\ elevation = tan^{-1}\left(R_{xy}/z_t - z_0\right) \quad (55)$$

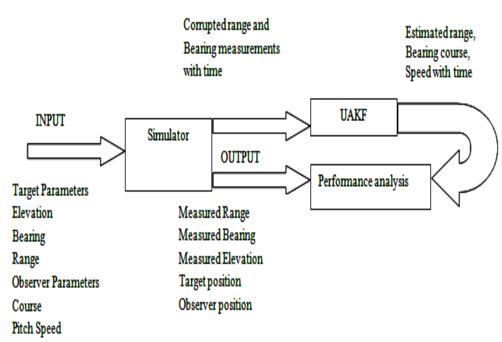


Fig. 3 Target motion analysis block diagram in simulation mode

The block diagram for the simulation mode target motion analysis is shown in Fig.3. Using EKF, the corrupted observations are used to estimate target motion parameters (TMPs). Estimated TMPs are related to true values.

EKF Algorithm:

EKF implementation is as follows.

i). The initial state vector estimate and its covariance matrix estimate be taken as $X(0|0)$ and $P(0|0)$ respectively.

ii). For the subsequent time, the state vector is calculated as $X_s(g+1)$ in (2)

iii). State vector's covariance matrix for the subsequent time is given as follows.

$$P(g+1|g) = \emptyset(g+1|g)P(g)\emptyset^T(g+1|g) + S(g+1) \quad (56)$$

iv). Gain of the EKF is considered as follows:

$$G(g+1) = P(g+1|g)\emptyset^T(g+1|g)[H(g+1)P(g+1|g)\,H^T(g+1) + R]^{-1} \quad (57)$$

v). The state estimation and its error covariance:

$$X_s(g+1|(g+1)) = X_s\,(g+1|g) + G(g+1)\,[Z(g+1) - \hat{Z}(g+1)] \quad (58)$$

$$P(g+1|g+1) = [1 - G(g+1)H(g+1)P(g+1|g)] \quad (59)$$

vi). For next iteration

$$X_s(g|g) = X(g+1|g+1) \quad (60)$$

$$P(g|g) = P(g+1|g+1) \quad (61)$$

III. Simulation and Results

The experiment is believed to be performed in ideal conditions. This simulation process is carried out via Matlab on a workstation. The trajectory chosen for algorithm evaluation is showed in Table.1. Scenario 1, for instance, defines a target moving within an opening range of 3000 m, with bearing and elevations of 45°. Its initial course is 255° moving with a speed of 10m/s. The range observations are tarnished with 10m (1σ), elevation and bearing measurements with, 0.33° (1σ) each.

TABLE I Input Scenarios chosen for the Algorithm

Parameter	Scenario	
	1	*2*
Target initial range (m)	3000	3000
Target initial bearing (deg)	45	135
Target initial course (deg)	255	315
Target initial speed (m/s)	10	8.5
Target initial Elevation (deg)	45	135
Bearing noise (1σ) (deg)	0.33	0.33
Range noise (1σ) (m)	10	10
Elevation angle noise (1 σ) (deg)	0.33	0.33

On the basis of weapon specification, an acceptance criterion is selected to sustain weapons (this issue is not discussed here) and is as follows. The solution is converged when inaccuracy in course <= 3°, inaccuracy in speed estimate <= 1m/s and inaccuracy in elevation estimate <= 1°. The target's estimated and real paths are as shown in Fig.4 and Fig.8 for scenario1 and 2 respectively. For clarity of the concepts, errors in predicted values of target speed, target course and target elevation for scenario1 are presented in Fig. 5, 6 and 7 respectively. For scenario 2 errors in speed, course and elevation are presented in Fig.9, 10 and 11 respectively. The solution is said to be obtained when the errors in estimated course, speed and elevation of the target are subject to the acceptance criteria. Table.2 provides the solution convergence time obtained in seconds for all predicted target parameters of scenarios in Table.1. It can be perceived from Table 2 data that the estimated parameters, i.e., course, speed and elevation of the target are converged at 28th, 26th and 3rd second respectively for scenario1. So, the total convergence of the solution is said to be obtained after 28 seconds. Similarly, for scenario 2, the convergence times of target parameters are 13th, 25th and 3rd seconds respectively for the estimated course, speed and elevation. So, the total convergence time of solution for scenario 2 is obtained after 25 seconds.

TABLE II Convergence Time For The Chosen Scenarios In Seconds

Parameter	Course		Speed		Elevation		Total solution	
Scenario	1	2	1	2	1	2	1	2
EKF	31	25	28	30	3	3	31	28
UKF	28	13	25	25	3	3	28	25

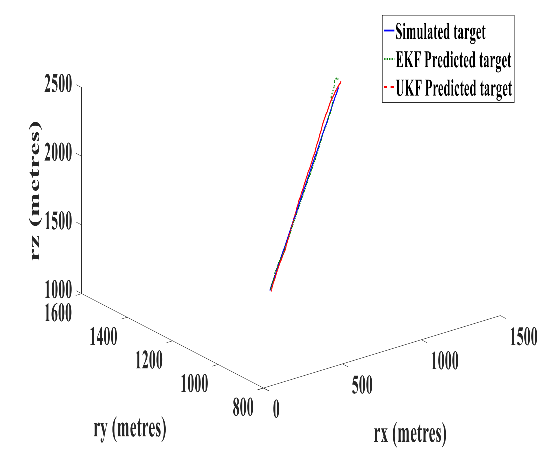


Fig. 4 Simulated and estimated target paths for scenario 1

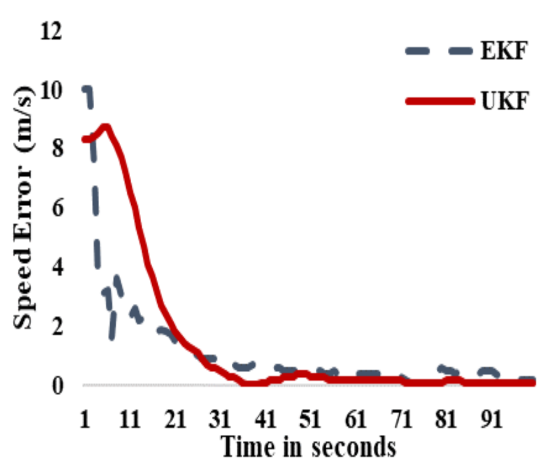


Fig. 5 Error in target speed estimate for scenario 1

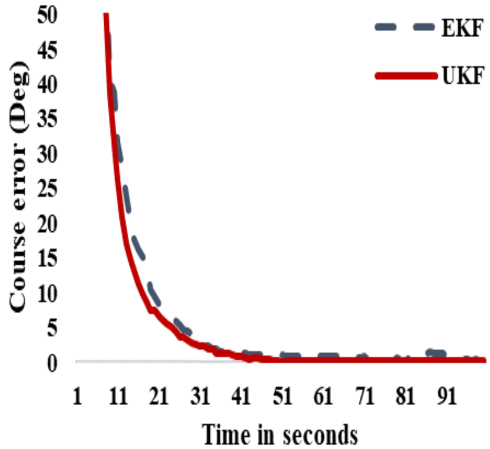


Fig. 6 Error in target course estimate for scenario 1

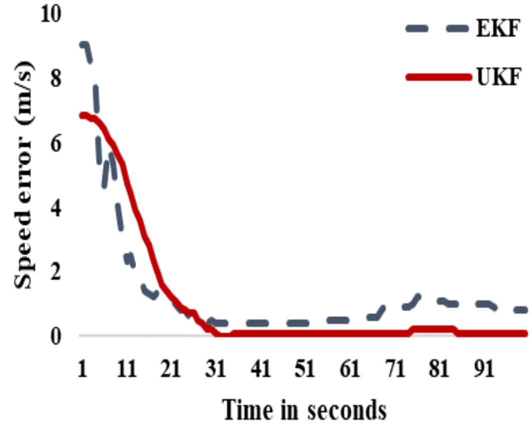


Fig.9. Error in target speed estimate for scenario 2

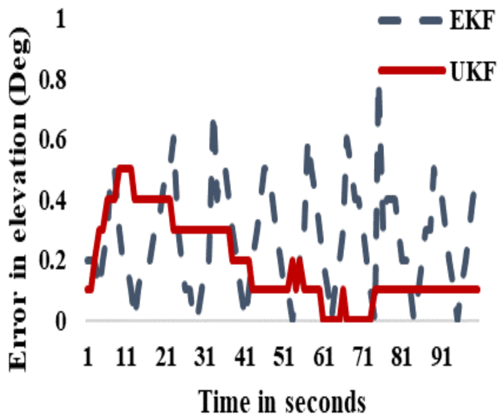


Fig.7. Error in target elevation estimate for scenario 1

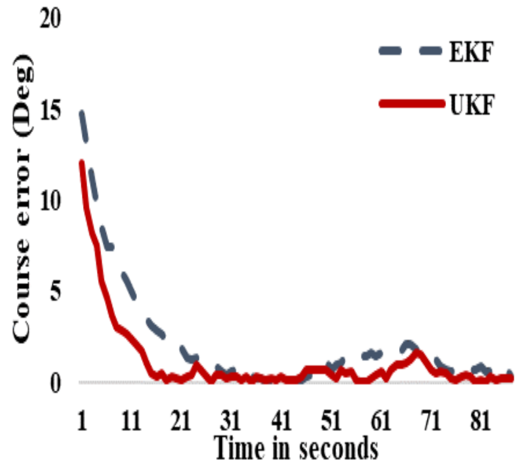


Fig.10. Error in target course estimate for scenario 2

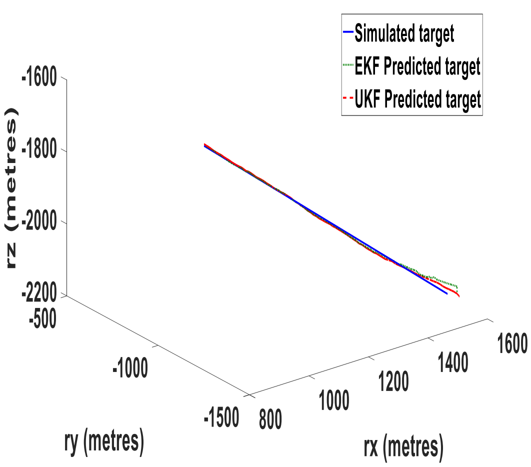


Fig.8. Simulated and estimated target paths for scenario 2

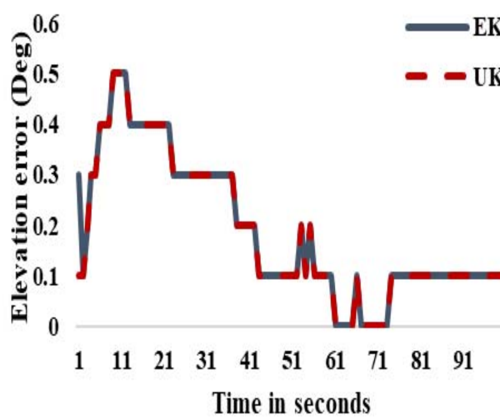


Fig.11. Error in target elevation estimate for scenario 2

From Table 2 it can be observed that the total convergence times of the solution is obtained earlier with UKF than EKF. As the tracking is carried out in active mode, the solution has to be obtained faster in order to take proper action on the target. It can also be observed from the figures that the error in estimated parameters is more using EKF than UKF. Moreover, EKF fails in many scenarios with higher nonlinearity. So, UKF is preferred for the application.

IV. Conclusion

In this research, an attempt is made to track the target in 3-D space using three different types of measurements. Unscented Kalman filter is preferred to predict target direction, speed in active target tracking from AAV systems. The results are obtained below 30 seconds. Hence, based on the results obtained during simulation UKF is suggested for active target tracking using AAV.

V. Futurescope

The research presented in this paper can be extended to maneuvering target tracking. The application can be explored with more nonlinear filtering algorithms like particle filter, shifted Rayleigh filter, ensemble Kalman filter, etc.

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